

Higgs Decays into Charginos and Neutralinos in the Complex MSSM: A Full One-Loop Analysis

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Abstract

For the search for additional Higgs bosons in the Minimal Supersymmetric Standard Model (MSSM) as well as for future precision analyses in the Higgs sector a precise knowledge of their decay properties is mandatory. We evaluate all two-body decay modes of the Higgs bosons into charginos and neutralinos in the MSSM with complex parameters (cMSSM). The evaluation is based on a full one-loop calculation of all decay channels, also including hard QED radiation. We restricted ourselves to a version of our renormalization scheme which is valid for $|M_1| < |M_2|, |\mu|$ and $M_2 \neq \mu$ to simplify the analysis, even though we are able to switch to other parameter regions. The dependence of the Higgs boson predictions on the relevant cMSSM parameters is analyzed numerically. We find sizable contributions to many partial decay widths. They are roughly of 10% of the tree-level results, but can go up to 20% or higher. The full one-loop contributions are important for the correct interpretation of heavy Higgs boson search results at the LHC and, if kinematically allowed, at a future linear e^+e^- collider. It is planned to implement the evaluation of the branching ratios of the Higgs bosons into the Fortran code **FeynHiggs**, together with an automated choice of the renormalization scheme valid for the full cMSSM parameter space.

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1 Introduction

One of the most important tasks at the LHC is to search for physics effects beyond the Standard Model (SM), where the Minimal Supersymmetric Standard Model (MSSM) [1–3] is one of the leading candidates. Supersymmetry (SUSY) predicts two scalar partners for all SM fermions as well as fermionic partners to all SM bosons. Another important task of the LHC is the investigation of the mechanism of electroweak symmetry breaking. The most frequently studied realizations are the Higgs mechanism within the SM and within the MSSM. Contrary to the case of the SM, in the MSSM two Higgs doublets are required. This results in five physical Higgs bosons instead of the single Higgs boson in the SM. In lowest order these are the light and heavy \mathcal{CP} -even Higgs bosons, h and H , the \mathcal{CP} -odd Higgs boson, A , and two charged Higgs bosons, H^\pm . Within the MSSM with complex parameters (cMSSM), taking higher-order corrections into account, the three neutral Higgs bosons mix and result in the states h_i ($i = 1, 2, 3$) [4–7]. The Higgs sector of the cMSSM is described at the tree-level by two parameters: the mass of the charged Higgs boson, M_{H^\pm} , and the ratio of the two vacuum expectation values, $\tan\beta \equiv t_\beta = v_2/v_1$. Often the lightest Higgs boson, h_1 is identified with the particle discovered at the LHC [8,9] with a mass around ~ 125 GeV. If the mass of the charged Higgs boson is assumed to be larger than ~ 200 GeV the four additional Higgs bosons are roughly mass degenerate, $M_{H^\pm} \approx m_{h_2} \approx m_{h_3}$ and referred to as the “heavy Higgs bosons”. Discovering one or more of those additional Higgs bosons would be an unambiguous sign of physics beyond the SM and could yield important information about their possible supersymmetric origin.

If SUSY is realized in nature and the charged Higgs-boson mass is $M_{H^\pm} \lesssim 1.5$ TeV, then the heavy Higgs bosons could be detectable at the LHC [10,11] (including its high luminosity upgrade, HL-LHC, see Ref. [12] and references therein) and/or at a future linear e^+e^- collider such as the ILC [13–15] or CLIC [16]. (Results on the combination of LHC and ILC results can be found in Ref. [17].) The discovery potential at the HL-LHC goes up to $\mathcal{O}(1$ TeV) for large $\tan\beta$ values and somewhat lower at low $\tan\beta$ values. At an e^+e^- linear collider the heavy Higgs bosons are pair produced, and the reach is limited by the center of mass energy, $M_{H^\pm} \lesssim \sqrt{s}/2$, roughly independent of $\tan\beta$. Details about the discovery process(es) depend strongly on the cMSSM parameters (and will not be further discussed in this paper).

In the case of a discovery of additional Higgs bosons a subsequent precision measurement of their properties will be crucial to determine their nature and the underlying (SUSY) parameters. In order to yield a sufficient accuracy, one-loop corrections to the various Higgs-boson decay modes have to be considered. Decays to SM fermions have been evaluated at the full one-loop level in the cMSSM in Ref. [18], see also Ref. [19] as well as Refs. [20,21] for higher-order SUSY corrections. Decays to (lighter) Higgs bosons have been evaluated at the full one-loop level in the cMSSM in Ref. [18], see also Refs. [22,23]. Decays to SM gauge bosons (see also Ref. [24]) can be evaluated to a very high precision using the full SM one-loop result [25] combined with the appropriate effective couplings [26]. The full one-loop corrections in the cMSSM listed here together with resummed SUSY corrections have been implemented into the code **FeynHiggs** [26–30]. Corrections at and beyond the one-loop level in the MSSM with real parameters (rMSSM) are implemented into the code **HDECAY** [31,32]. Both codes were combined by the LHC Higgs Cross Section Working Group to obtain the

most precise evaluation for rMSSM Higgs boson decays to SM particles and decays to lighter Higgs bosons [33].

The (heavy) MSSM Higgs bosons can (if kinematically allowed) also decay to SUSY particles, i.e. to charginos, neutralinos and scalar fermions. In Ref. [34] it was demonstrated that the SUSY particle modes can dominate the decay of the heavy Higgs bosons. The lightest neutral Higgs boson, on the other hand, can have a substantial branching ratio into the lightest neutralino, $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$, where the $\tilde{\chi}_1^0$ constitutes the Dark Matter candidate in the MSSM [35]. Bounds on $m_{\tilde{\chi}_1^0}$ often assume an underlying SUSY grand unified theory, based on a simple Lie group. Dropping these assumptions hardly any bound on $m_{\tilde{\chi}_1^0}$ can be placed directly (see, e.g., Ref. [36] and references therein), and the decay $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ is kinematically possible. In order to determine the Dark Matter properties a precision measurement of this process at the LHC or a future e^+e^- collider will be necessary.

Higher-order contributions to MSSM Higgs boson decays to scalar fermions have been evaluated in various analyses over the last decade. For calculations in the rMSSM, see Refs. [37–39] and references therein. More recently, the results of Ref. [37] were made public in the code **HFOLD** [40], using a pure $\overline{\text{DR}}$ renormalization for the calculation. In Ref. [41] the $\mathcal{O}(\alpha_s)$ corrections to Higgs boson decays to scalar quarks were re-analyzed and included into the code **HDECAY**. Within the cMSSM a full one-loop calculation of Higgs boson decays to scalar fermions has recently been published in Ref. [42] and will be included into the code **FeynHiggs**. These results were obtained in a renormalization scheme [26, 43–48], which has been shown to yield stable results over nearly the full cMSSM parameters space. In this work we take another step in the direction of completion of the calculation of *all* two-body decays at the one-loop level in the cMSSM in this stable and reliable renormalization scheme: we calculate all two-body decay modes of the Higgs bosons to charginos and neutralinos in the cMSSM. More specifically, we calculate the full one-loop corrections to the partial decay widths

$$\Gamma(h_i \rightarrow \tilde{\chi}_c^- \tilde{\chi}_{c'}^+) \quad (i = 1, 2, 3; c, c' = 1, 2), \quad (1)$$

$$\Gamma(h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0) \quad (i = 1, 2, 3; n, n' = 1, 2, 3, 4), \quad (2)$$

$$\Gamma(H^\pm \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_c^\pm) \quad (n = 1, 2, 3, 4; c = 1, 2), \quad (3)$$

where $\tilde{\chi}_c^\pm$ ($\tilde{\chi}_n^0$) denotes the charginos (neutralinos). While we have calculated the decay of all Higgs bosons, in the numerical evaluation below, we will concentrate on the heavy Higgs bosons, $h_{2,3}$ and H^\pm , but also show results for $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$.

The evaluation of the channels Eqs. (1) – (3) is based on a full one-loop calculation, i.e. including electroweak (EW) corrections, as well as soft and hard QED radiation. For “mixed” decay modes, we evaluate in addition the two “ \mathcal{CP} -versions” ($c \neq c'$) of Eq. (1) and the two “ \mathcal{CP} -versions” of Eq. (3), which give different results for non-zero complex phases. We restricted ourselves to a version of our renormalization scheme which is valid for $|M_1| < |M_2|, |\mu|$ and $M_2 \neq \mu$ (where M_1 and M_2 denote the soft SUSY-breaking parameter of the $U(1)$ and $SU(2)$ gauginos, and μ is the Higgs mixing parameter) to simplify the analysis, even though we are able to switch to other parameter regions, see the discussion in Refs. [46–48] (see also Ref. [49]).

Higher-order contributions to MSSM Higgs boson decays to charginos and neutralinos have been evaluated in various analyses over the last decade. In Ref. [50] the leading Yukawa

corrections to $A/H \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0, \tilde{\chi}_1^+ \tilde{\chi}_1^-$ in the rMSSM have been evaluated, employing an on-shell (OS) scheme (referring to Ref. [51], but without providing further details). Next, in Ref. [52] the full one-loop corrections to $A/H \rightarrow \tilde{\chi}_c^+ \tilde{\chi}_{c'}^-$ ($c, c' = 1, 2$) have been presented in the rMSSM (again without details about the OS like scheme). An effective Lagrangian approach for heavy neutral Higgs boson decays in the rMSSM was published in Ref. [53]. The full one-loop corrections to all heavy Higgs decays to charginos and neutralinos in the rMSSM in the $\overline{\text{DR}}$ scheme was published in the code HFOLD [40]. More recently also evaluations of Higgs boson decays to charginos and neutralinos in the cMSSM became available. In Ref. [54] the decays $h_i \rightarrow \tilde{\chi}_c^+ \tilde{\chi}_{c'}^-$, ($i = 1, 2, 3$; $c, c' = 1, 2$) were presented, together with a short discussion of different renormalization schemes (see Sect. 2.2) and brief analysis of the dependence on the phases of μ , M_1 and the trilinear Higgs stop coupling, A_t . The decays $h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0$ ($i = 1, 2, 3$; $n, n' = 1, 2, 3, 4$) were calculated in Ref. [55], where the numerical analysis concentrated on $h_{2,3} \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$ and the dependence on the phase of A_t . The latter two references are close to the calculations presented in this paper. Small differences in the renormalization in the chargino/neutralino sector exist (see Sect. 2.2 and Ref. [47]), where we use consistently the scheme detailed in Ref. [45] for *all* two-body decays *simultaneously*. In our numerical analysis we focus on the one parameter with a possible complex phase entering at the tree-level, M_1 , see the discussion in Sect. 3. A short numerical comparison with the literature, in particular with Refs. [40, 54, 55], will be given in Sect. 4.1.

In this paper we present a full one-loop calculation for all two-body decay channels of the Higgs bosons into charginos and neutralinos in the cMSSM, taking into account soft and hard QED radiation. In Sect. 2 we briefly review the relevant sectors of the cMSSM. Details about the calculation can be found in Sect. 3, and the numerical results for all decay channels are presented in Sect. 4 (including comments on comparisons with results from other groups). The conclusions can be found in Sect. 5. It is planned to implement the evaluation of the branching ratios of the heavy Higgs bosons into the Fortran code `FeynHiggs` [26–30], together with an automated choice of the renormalization scheme valid for the full cMSSM parameter space.

2 The complex MSSM

The channels (1) – (3) are calculated at the one-loop level, including soft and hard QED radiation. This requires the simultaneous renormalization of several sectors of the cMSSM: the Higgs and gauge boson sector as well as the chargino/neutralino sector. In the following subsections we very briefly review these sectors and their renormalization.

2.1 The Higgs- and gauge-boson sector

The Higgs- and gauge-boson sector follow strictly Ref. [45] and references therein (see especially Ref. [26]). This defines in particular the counterterm $\delta \tan \beta \equiv \delta t_\beta$, as well as the counterterms for the Z boson mass, δM_Z^2 , and for the sine of the weak mixing angle, δs_w (with $s_w = \sqrt{1 - c_w^2} = \sqrt{1 - M_W^2/M_Z^2}$, where M_W denotes the W boson mass).

2.2 The chargino/neutralino sector

The chargino/neutralino sector is also described in detail in Ref. [45] and references therein, see in particular Refs. [46–48]. In this paper we use the so called “CCN” scheme, i.e. OS conditions for two charginos and one neutralino, which we chose to be the lightest one. Renormalizing the two charged states OS, i.e. ensuring that they have the same mass at the tree- and at the loop-level is (in general) crucial for the cancellation of the IR divergencies. In the notation of Ref. [45] we used:

$$\text{\$InoScheme} = \text{CCN}[1] \quad \text{fixed CCN scheme with on-shell } \tilde{\chi}_1^0.$$

This defines in particular the counterterm $\delta\mu$, where μ denotes the Higgs mixing parameter. This scheme yields numerically stable results for $|M_1| < |M_2|, |\mu|$ and $M_2 \neq \mu$, i.e. the lightest neutralino is bino-like and defines the counterterm for M_1 [46–49]. In the numerical analysis this mass pattern holds. Switching to a different mass pattern, e.g. with $|M_2| < |M_1|$ and/or $M_2 \sim \mu$ requires to switch to a different renormalization scheme [45, 49]. While these schemes are implemented into the **FeynArts/FormCalc** framework [45], so far no automated choice of the renormalization scheme has been devised. For simplicity we stick to the CCN[1] scheme with a matching choice of SUSY parameters, see Sect. 4.2.

Since both chargino masses $m_{\tilde{\chi}_{1,2}^\pm}$ and the lightest neutralino mass $m_{\tilde{\chi}_1^0}$ have been chosen as independent parameters, the one-loop masses of the heavier neutralinos $\tilde{\chi}_n^0$ ($n = 2, 3, 4$) are obtained from the tree-level ones via the shifts [55]

$$\begin{aligned} \Delta m_{\tilde{\chi}_n^0} = -\text{Re} \left\{ m_{\tilde{\chi}_n^0} \left(\Sigma_{\tilde{\chi}_n^0}^L(m_{\tilde{\chi}_n^0}^2) + \frac{1}{2} \left[\delta \mathbf{Z}_{\tilde{\chi}^0}^L + \delta \check{\mathbf{Z}}_{\tilde{\chi}^0}^L + \delta \mathbf{Z}_{\tilde{\chi}^0}^R + \delta \check{\mathbf{Z}}_{\tilde{\chi}^0}^R \right]_{nn} \right) \right. \\ \left. + \Sigma_{\tilde{\chi}_n^0}^{SL}(m_{\tilde{\chi}_n^0}^2) - m_{\tilde{\chi}_n^0} \left[\delta \mathbf{Z}_{\tilde{\chi}^0}^L + \delta \check{\mathbf{Z}}_{\tilde{\chi}^0}^L \right]_{nn} - [\delta \mathbf{M}_{\tilde{\chi}^0}]_{nn} \right\}, \end{aligned} \quad (4)$$

where the renormalization constants $\delta \mathbf{Z}$ and $\delta \check{\mathbf{Z}}$ can be found in Ref. [45]. For all externally appearing neutralino masses we use the (shifted) “on-shell” masses:

$$m_{\tilde{\chi}_n^0}^{\text{os}} = m_{\tilde{\chi}_n^0} + \Delta m_{\tilde{\chi}_n^0}. \quad (5)$$

In order to yield UV-finite results we use the tree-level values $m_{\tilde{\chi}_n^0}$ for all internally appearing neutralino masses in loop calculations.

2.3 The fermion/sfermion sector

To be in accordance with Ref. [42], we use shifted (s)fermion masses in the loop corrections. As requirement for these shifts one needs the renormalization of the fermion/sfermion sector:

- The renormalization of the fermion sector is described in detail in Ref. [45] and references therein. For simplification we use the $\overline{\text{DR}}$ renormalization for all three generations of down-type quarks *and* leptons, again in the notation of Ref. [45]:

$$\begin{aligned} \text{UVMf1}[4, _] &= \text{UVDivergentPart} & \overline{\text{DR}} \text{ renormalization for } m_d, m_s, m_b \\ \text{UVMf1}[2, _] &= \text{UVDivergentPart} & \overline{\text{DR}} \text{ renormalization for } m_e, m_\mu, m_\tau \end{aligned}$$

- The renormalization of the sfermion sector differs slightly from the one described in Ref. [45]. For the squark sector we follow Refs. [43, 44] (which agrees with the renormalization scheme used in Refs. [46, 47, 56, 57]) and the slepton sector can be found in Ref. [42]. Concerning our notation we denote as $M_{\tilde{Q}_g, \tilde{L}_g, \tilde{U}_g, \tilde{D}_g, \tilde{E}_g}$, the “diagonal” soft SUSY-breaking parameters for the $SU(2)$ squark, slepton doublet, the u-, d-type squark singlet, and the e-type slepton singlet, respectively, where g is the generation index. Furthermore we use A_{u_g, d_g, e_g} for the trilinear Higgs-scalar u-, d-, e-type fermion couplings, respectively.

3 Calculation of loop diagrams

In this section we give some details about the calculation of the higher-order corrections to the partial decay widths of Higgs bosons. Sample diagrams for the decays $h_i \rightarrow \tilde{\chi}_c^- \tilde{\chi}_c^+$ ($i = 1, 2, 3$; $c, c' = 1, 2$), $h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0$ ($i = 1, 2, 3$; $n, n' = 1, 2, 3, 4$) and $H^+ \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_c^+$ ($n = 1, 2, 3, 4$; $c = 1, 2$) are shown in Figs. 1, 2 and 3, respectively. Not shown are the diagrams for real (hard and soft) photon radiation. They are obtained from the corresponding tree-level diagrams by attaching a photon to the electrically charged particles. The internal generically depicted particles in Figs. 1, 2 and 3 are labeled as follows: F can be a SM fermion f , chargino $\tilde{\chi}_c^\pm$, neutralino $\tilde{\chi}_n^0$; S can be a sfermion \tilde{f}_s or a Higgs (Goldstone) boson h_i (G); V can be a photon γ or a massive SM gauge boson, Z or W^\pm . For internally appearing Higgs bosons no higher-order corrections to their masses or couplings are taken into account; these corrections would correspond to effects beyond one-loop order.¹ For external Higgs bosons, as discussed in Ref. [26], the appropriate \hat{Z} factors are applied and OS masses (including higher-order corrections) are used [26], obtained with **FeynHiggs** [26–30].

Also not shown are the diagrams with a Higgs boson–gauge/Goldstone self-energy contribution on the external Higgs boson leg. They appear in the decay $h_i \rightarrow \tilde{\chi}_c^- \tilde{\chi}_c^+$, Fig. 1 and $h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0$, Fig. 2, with a h_i – Z/G transition and in the decay $H^\pm \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_c^\pm$, Fig. 3, with a H^\pm – W^\pm/G^\pm transition.²

Furthermore, in general, in Figs. 1 – 3 we have omitted diagrams with self-energy type corrections of external (on-shell) particles. While the contributions from the real parts of the loop functions are taken into account via the renormalization constants defined by OS renormalization conditions, the contributions coming from the imaginary part of the loop functions can result in an additional (real) correction if multiplied by complex parameters. In the analytical and numerical evaluation, these diagrams have been taken into account via the prescription described in Ref. [45].

Within our one-loop calculation we neglect finite width effects that can help to cure threshold singularities. Consequently, in the close vicinity of those thresholds our calculation does not give a reliable result. Switching to a complex mass scheme [58] would be another possibility to cure this problem, but its application is beyond the scope of our paper.

The diagrams and corresponding amplitudes have been obtained with **FeynArts** [59]. The model file, including the MSSM counterterms, is largely based on Ref. [45], however

¹ We found that using loop corrected Higgs boson masses in the loops leads to a UV divergent result.

² From a technical point of view, the H^\pm – W^\pm/G^\pm transitions have been absorbed into the respective counterterms, while the h_i – Z/G transitions have been calculated explicitly.

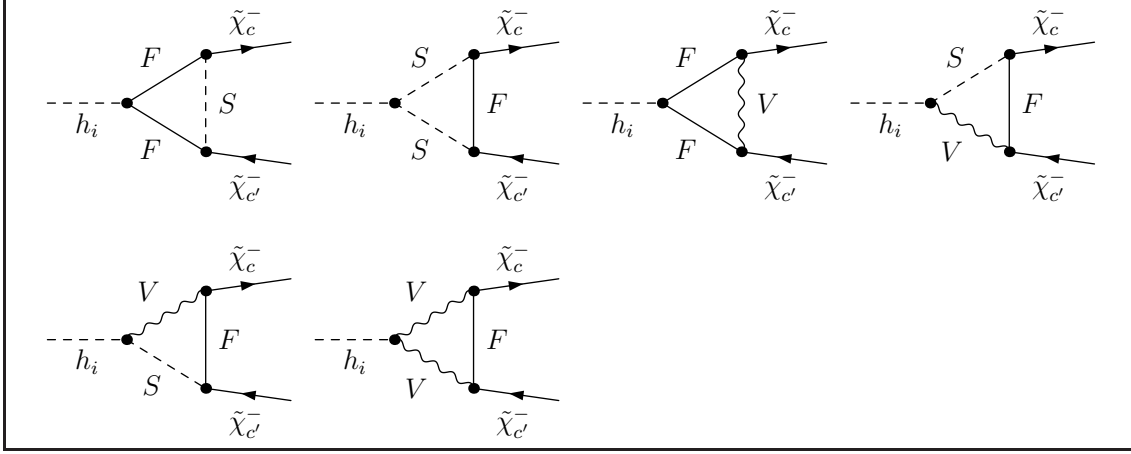


Figure 1: Generic Feynman diagrams for the decay $h_i \rightarrow \tilde{\chi}_c^- \tilde{\chi}_{c'}^+$ ($i = 1, 2, 3$; $c, c' = 1, 2$). F can be a SM fermion, chargino, neutralino; S can be a sfermion or a Higgs/Goldstone boson; V can be a γ , Z , W^\pm . Not shown are the diagrams with a h_i - Z or h_i - G transition contribution on the external Higgs boson leg.

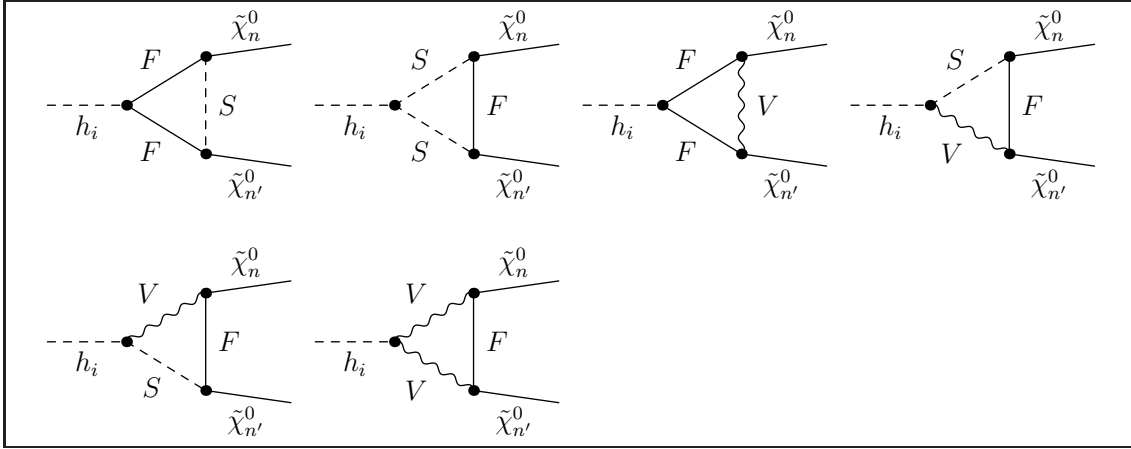


Figure 2: Generic Feynman diagrams for the decay $h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0$ ($i = 1, 2, 3$; $n, n' = 1, 2, 3, 4$). F can be a SM fermion, chargino, neutralino; S can be a sfermion or a Higgs/Goldstone boson; V can be a Z or W^\pm . Not shown are the diagrams with a h_i - Z or h_i - G transition contribution on the external Higgs boson leg.

adjusted to match exactly the renormalization prescription described in Sect. 2. The further evaluation has been performed with **FormCalc** and **LoopTools** [60].

Ultraviolet divergences

As regularization scheme for the UV divergences we have used constrained differential renormalization [61], which has been shown to be equivalent to dimensional reduction [62] at the one-loop level [60]. Thus the employed regularization scheme preserves SUSY [63, 64] and

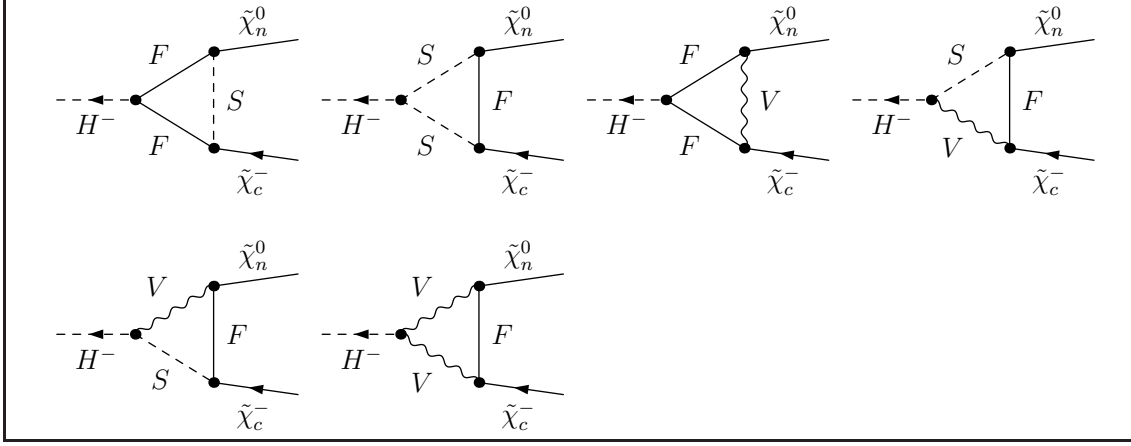


Figure 3: Generic Feynman diagrams for the decay $H^+ \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_c^+$ ($n = 1, 2, 3, 4; c = 1, 2$). (It should be noted that all arrows are inverted in case of a H^- decay.) F can be a SM fermion, chargino, neutralino; S can be a sfermion or a Higgs/Goldstone boson; V can be a γ , Z , W^\pm . Not shown are the diagrams with a $H^\pm - W^\pm$ or $H^\pm - G^\pm$ transition contribution on the external Higgs boson leg.

guarantees that the SUSY relations are kept intact, e.g. that the gauge couplings of the SM vertices and the Yukawa couplings of the corresponding SUSY vertices also coincide to one-loop order in the SUSY limit. Therefore no additional shifts, which might occur when using a different regularization scheme, arise. All UV divergences cancel in the final result.

Infrared divergences

The IR divergences from diagrams with an internal photon have to cancel with the ones from the corresponding real soft radiation. In the case of QED we have included the soft photon contribution following the description given in Ref. [51]. The IR divergences arising from the diagrams involving a γ are regularized by introducing a photon mass parameter, λ . All IR divergences, i.e. all divergences in the limit $\lambda \rightarrow 0$, cancel once virtual and real diagrams for one decay channel are added.

Tree-level formulas

For completeness we show here also the formulas that have been used to calculate the tree-level decay widths:

$$\Gamma^{\text{tree}}(H^\pm \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_c^\pm) = \left[(|C(H^\pm, \tilde{\chi}_n^0, \tilde{\chi}_c^\pm)_L|^2 + |C(H^\pm, \tilde{\chi}_n^0, \tilde{\chi}_c^\pm)_R|^2) (M_{H^\pm}^2 - m_{\tilde{\chi}_n^0}^2 - m_{\tilde{\chi}_c^\pm}^2) - 4 \operatorname{Re} \{ C(H^\pm, \tilde{\chi}_n^0, \tilde{\chi}_c^\pm)_L^* C(H^\pm, \tilde{\chi}_n^0, \tilde{\chi}_c^\pm)_R \} m_{\tilde{\chi}_n^0} m_{\tilde{\chi}_c^\pm} \right] \times \frac{\lambda^{1/2} (M_{H^\pm}^2, m_{\tilde{\chi}_n^0}^2, m_{\tilde{\chi}_c^\pm}^2)}{16 \pi M_{H^\pm}^3} \quad (c = 1, 2; n = 1, 2, 3, 4), \quad (6)$$

$$\Gamma^{\text{tree}}(h_i \rightarrow \tilde{\chi}_c^- \tilde{\chi}_{c'}^+) = \left[(|C(h_i, \tilde{\chi}_c^-, \tilde{\chi}_{c'}^+)_L|^2 + |C(h_i, \tilde{\chi}_c^-, \tilde{\chi}_{c'}^+)_R|^2) (m_{h_i}^2 - m_{\tilde{\chi}_c^-}^2 - m_{\tilde{\chi}_{c'}^+}^2) - 4 \operatorname{Re} \{ C(h_i, \tilde{\chi}_c^-, \tilde{\chi}_{c'}^+)_L^* C(h_i, \tilde{\chi}_c^-, \tilde{\chi}_{c'}^+)_R \} m_{\tilde{\chi}_c^-} m_{\tilde{\chi}_{c'}^+} \right] \times \frac{\lambda^{1/2}(m_{h_i}^2, m_{\tilde{\chi}_c^-}^2, m_{\tilde{\chi}_{c'}^+}^2)}{16 \pi m_{h_i}^3} \quad (i = 1, 2, 3; c, c' = 1, 2), \quad (7)$$

$$\Gamma^{\text{tree}}(h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0) = \left[(|C(h_i, \tilde{\chi}_n^0, \tilde{\chi}_{n'}^0)_L|^2 + |C(h_i, \tilde{\chi}_n^0, \tilde{\chi}_{n'}^0)_R|^2) (m_{h_i}^2 - m_{\tilde{\chi}_n^0}^2 - m_{\tilde{\chi}_{n'}^0}^2) - 4 \operatorname{Re} \{ C(h_i, \tilde{\chi}_n^0, \tilde{\chi}_{n'}^0)_L^* C(h_i, \tilde{\chi}_n^0, \tilde{\chi}_{n'}^0)_R \} m_{\tilde{\chi}_n^0} m_{\tilde{\chi}_{n'}^0} \right] \times \frac{\lambda^{1/2}(m_{h_i}^2, m_{\tilde{\chi}_n^0}^2, m_{\tilde{\chi}_{n'}^0}^2)}{16 \pi m_{h_i}^3} \quad (i = 1, 2, 3; n, n' = 1, 2, 3, 4), \quad (8)$$

where $\lambda(x, y, z) = (x - y - z)^2 - 4yz$. The couplings $C(a, b, c)$ can be found in the **FeynArts** model files, see Ref. [65]. $C(a, b, c)_{L,R}$ denote the part of the coupling which is proportional to $(\mathbb{1} \mp \gamma_5)/2$. For the later interpretation of the results in the numerical analysis the following should be kept in mind. In Eqs. (7), (8) the couplings of the Higgs to charginos/neutralinos result in a relative plus (minus) sign between the two terms (in the first and second line of each equation, respectively) for h_i being a \mathcal{CP} -odd (\mathcal{CP} -even) Higgs, leading to an enhancement (suppression) of the decay width. In case of equal final state masses ($c = c'$ or $n = n'$) one finds in **FeynArts** convention³ $C(h_i, \tilde{\chi}, \tilde{\chi})_L = -C(h_i, \tilde{\chi}, \tilde{\chi})_R^* =: C(h_i, \tilde{\chi}, \tilde{\chi})$, and the general structure of the tree-level decay width simplifies for *real* parameters to

$$\Gamma^{\text{tree}}(h_i \rightarrow \tilde{\chi} \tilde{\chi}) = \frac{|C(h_i, \tilde{\chi}, \tilde{\chi})|^2}{8 \pi} [m_{h_i}^2 - 4 m_{\tilde{\chi}}^2]^{(1/2)} \quad \text{for } h_i \text{ } \mathcal{CP}\text{-odd}, \quad (9)$$

$$\Gamma^{\text{tree}}(h_i \rightarrow \tilde{\chi} \tilde{\chi}) = \frac{|C(h_i, \tilde{\chi}, \tilde{\chi})|^2}{8 \pi m_{h_i}^2} [m_{h_i}^2 - 4 m_{\tilde{\chi}}^2]^{(3/2)} \quad \text{for } h_i \text{ } \mathcal{CP}\text{-even}. \quad (10)$$

The latter decay width exhibits a p -wave suppression.

4 Numerical analysis

In this section we present the comparisons with results from other groups and our numerical analysis of the light and heavy Higgs boson decay channels into charginos and neutralinos in the cMSSM. In the various figures below we show the partial decay widths and their relative correction at the tree-level (“tree”) and at the one-loop level (“full”).

4.1 Comparisons

We performed exhaustive comparisons with results from other groups for Higgs boson decays into charginos and neutralinos. Most of these comparisons were restricted to the MSSM with real parameters.

³ It should be noted that the convention for Feynman rules in Ref. [2] and in Ref. [65] differ by a global factor of $-i$, which would formally lead to $C(h_i, \tilde{\chi}, \tilde{\chi})_L = +C(h_i, \tilde{\chi}, \tilde{\chi})_R^*$. However, the physics outcome remains, of course, unchanged.

Table 1: Comparison of the 1-loop corrected partial decay widths (in GeV) with Ref. [53].

Process	Mass	Ref. [52]	Ref. [53]	FeynTools
$A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	$M_A = 700$	0.85	0.80	0.83
$A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	$M_A = 800$	1.00	0.91	0.96
$H^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	$M_H = 800$	0.63	0.58	0.64
$H^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	$M_H = 900$	0.73	0.70	0.75

- A comparison with Ref. [50] (in the rMSSM) gave an overall qualitative agreement for the decays $H/A \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, as was to be expected, because that work took into account only the leading Yukawa corrections and used a different renormalization scheme. On the other hand, we omit a comparison with the results for $H/A \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$ of Ref. [50], because in their set-up neutralino masses were used as input parameters, which is rather difficult to adapt to our numerical analysis set-up.
- For the comparison with Ref. [52] we calculated the decays $A/H \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ at $\mathcal{O}(\alpha(M_Z))$, using their input parameters as far as possible. We found good (qualitative) agreement with Ref. [52] (where the calculation was restricted to the rMSSM). We successfully reproduced their Figs. 2, 4, 5 and 6, where only a small difference remains due to the different renormalization schemes, see also Tab. 1 with differences below 5%.
- We performed a numerical comparison with Ref. [53] for the decay $H/A \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ at $\mathcal{O}(\alpha(M_Z))$ (in the rMSSM), see Tab. 1, where the columns for Ref. [52] and Ref. [53] were taken over from Tab. 1 of the first article in Ref. [53]. Their set-up differs from ours in the renormalization of the chargino/neutralino sector, leading to different loop corrections. Furthermore they used an “effective one-loop Lagrangian”. Nevertheless, using their input parameters as far as possible, we found differences below the 10% level.
- A numerical comparison with the program HFOLD Ref. [40] at the benchmark point SPS1a’ (proposed in the SPA project [66]) can be found in Tab. 2. Only for this point sufficient details about the chargino/neutralino masses was available for a numerical comparison. In Tab. 2 we show the full one-loop results of HFOLD, using $\overline{\text{DR}}$ masses for the internal and external particles, corresponding to the full $\overline{\text{DR}}$ renormalization used in the code (where the renormalization scale was set to 1 TeV [66]). Our results, labeled FeynTools, are evaluated using our renormalization scheme, but inserting the HFOLD $\overline{\text{DR}}$ masses. In the tree-level results we find more than 10 digits agreement and in the full results we find agreement of 3% – 15% (7% on average).
HFOLD also offers to switch to (the recommended) OS masses for the external particles. In this case, we are including in our calculation $m_{\tilde{\chi}_2^0}^{\text{os}}$ as described in Sect. 2.2, Eq. (5), but using the same OS Higgs boson masses as in HFOLD. With it the agreement between the two calculations is 11% on average (5% – 22%).

- Decays of $h_{2,3}$ to charginos in the cMSSM at the full one-loop level have been nu-

Table 2: Comparison of the 1-loop corrected partial decay widths (in 10^{-1} GeV) with HFOLD.

Process	OS masses		$\overline{\text{DR}}$ masses	
	HFOLD	FeynTools	HFOLD	FeynTools
$H^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$	0.1381	0.1648	0.1046	0.1229
$H^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$	0.4584	0.4908	0.2690	0.2828
$H^0 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$	0.2061	0.2259	0.0117	0.0111
$H^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	0.5262	0.5672	0.0345	0.0332
$A^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$	0.2044	0.2404	0.1704	0.2016
$A^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$	0.9693	1.0248	0.7334	0.7750
$A^0 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$	1.1652	1.0747	0.3966	0.3791
$A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$	2.8604	2.6454	1.0236	0.9928
$H^+ \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^+$	1.2981	1.4307	0.9333	0.9996
$H^+ \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^+$	0.0063	0.0081	0.0026	0.0030

merically compared with Ref. [54] using their latest **FeynArts/FormCalc** model file implementation. We found overall agreement better than 1% in the loop corrections for real and complex parameters.⁴

- $h_{2,3}$ boson decays into $\tilde{\chi}_2^0 \tilde{\chi}_2^0$ in the cMSSM have been analyzed in Ref. [55]. Again we had to use here the latest **FeynArts/FormCalc** model file implementation of Ref. [54] (which bases mainly on code from Ref. [55]) for the same reasons as described in the previous item. In comparison with that model file [54] we found overall agreement better than 2% in the loop corrections for real and complex parameters.

4.2 Parameter settings

The renormalization scale μ_R has been set to the mass of the decaying Higgs boson. The SM parameters are chosen as follows; see also [68]:

- Fermion masses (on-shell masses, if not indicated differently):

$$\begin{aligned}
m_e &= 0.510998928 \text{ MeV} , & m_{\nu_e} &= 0 \text{ MeV} , \\
m_\mu &= 105.65837515 \text{ MeV} , & m_{\nu_\mu} &= 0 \text{ MeV} , \\
m_\tau &= 1776.82 \text{ MeV} , & m_{\nu_\tau} &= 0 \text{ MeV} , \\
m_u &= 68.7 \text{ MeV} , & m_d &= 68.7 \text{ MeV} , \\
m_c &= 1.275 \text{ GeV} , & m_s &= 95.0 \text{ MeV} , \\
m_t &= 173.21 \text{ GeV} , & m_b &= 4.18 \text{ GeV} .
\end{aligned} \tag{11}$$

⁴ It should be noted that the original code used for Ref. [54] is no longer available [67], where we found some numerical differences with the results shown in Ref. [54] in the case of complex parameters.

According to Ref. [68], m_s is an estimate of a so-called "current quark mass" in the $\overline{\text{MS}}$ scheme at the scale $\mu \approx 2$ GeV. $m_c \equiv m_c(m_c)$ and $m_b \equiv m_b(m_b)$ are the "running" masses in the $\overline{\text{MS}}$ scheme.⁵ m_u and m_d are effective parameters, calculated through the hadronic contributions to

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = \frac{\alpha}{\pi} \sum_{f=u,c,d,s,b} Q_f^2 \left(\ln \frac{M_Z^2}{m_f^2} - \frac{5}{3} \right) \sim 0.027723. \quad (12)$$

- Gauge boson masses:

$$M_Z = 91.1876 \text{ GeV}, \quad M_W = 80.385 \text{ GeV}. \quad (13)$$

- Coupling constant:

$$\alpha(0) = 1/137.0359895. \quad (14)$$

The Higgs sector quantities (masses, mixings, etc.) have been evaluated using **FeynHiggs** (version 2.10.2) [26–30].

We emphasize again that the analytical calculation has been done for *all* decays into charginos/neutralinos. Results are shown for some representative numerical examples. The parameters are chosen according to the scenarios, SX ($X = 1, 2, \dots, 5$), shown in Tab. 3, unless otherwise noted. The scenarios are defined such that a maximum number of decay modes are open simultaneously to permit an analysis of all channels, i.e. not picking specific parameters for each decay. For the same reason we do not demand that the lightest Higgs boson has a mass around ~ 125 GeV, although for most of the parameter space this is given. For the light Higgs we will show the variation with M_{H^\pm} , $|\mu|$, M_1 and φ_{M_1} (where the latter denotes the phase of the gaugino mass parameter M_1), whereas for the heavy Higgs bosons we will analyze the variation of M_{H^\pm} and φ_{M_1} .

The numerical results shown in the next subsections are of course dependent on choice of the SUSY parameters. Nevertheless, they give an idea of the relevance of the full one-loop corrections. Channels (and their respective one-loop corrections) that may look unobservable due to the smallness of their decay width in the plots shown below, could become important if other channels are kinematically forbidden.

4.3 Full one-loop results for varying M_{H^\pm} , M_1 , and φ_{M_1}

The results shown in this and the following subsections consist of "tree", which denotes the tree-level value and of "full", which is the partial decay width including *all* one-loop corrections as described in Sect. 3. We restrict ourselves to the analysis of the decay widths themselves, since the one-loop effects on the branching ratios are strongly parameter dependent, as discussed in the previous subsection.

When performing an analysis involving complex parameters it should be noted that the results for physical observables are affected only by certain combinations of the complex phases of the parameters μ , the trilinear couplings A_f and the gaugino mass parameters

⁵ It should be noted, that in the analysis below, we use the $\overline{\text{DR}}$ mass $m_b^{\overline{\text{DR}}}$ from Eq.(19) of Ref. [42].

Table 3: MSSM parameters for the initial numerical investigation; all parameters (except of t_β) are in GeV (calculated masses are rounded to 1 MeV). In our analysis $M_{\tilde{Q}_3}$, $M_{\tilde{U}_3}$, $M_{\tilde{D}_3}$, $M_{\tilde{L}_3}$ and $M_{\tilde{E}_3}$ are chosen such that the values of $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $m_{\tilde{b}_2}$, $m_{\tilde{\nu}_\tau}$ and $m_{\tilde{\tau}_2}$ are realized. For the sfermion sector the shifts in $M_{\tilde{Q},\tilde{D}}(\tilde{d}_g)$ and $M_{\tilde{L},\tilde{E}}(\tilde{e}_g)$ as defined in Ref. [42] are taken into account. The values for the trilinear sfermion Higgs couplings, A_{u_g,d_g,e_g} ($g = 1, 2, 3$; identical for all g) are chosen such that charge- and/or color-breaking minima are avoided [69]. It should be noted that for the first and second generation of sfermions we chose instead $M_{\tilde{L},\tilde{E}} = 1500$ GeV and $M_{\tilde{Q},\tilde{U},\tilde{D}} = 2000$ GeV. For the neutralino sector the shifts in Eq. (4) are taken into account.

Scen.	t_β	μ	A_{u_g}	A_{d_g}	A_{e_g}	$ M_1 $	M_2	M_3	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_2}$	$m_{\tilde{\nu}_\tau}$	$m_{\tilde{\tau}_2}$
SX	10	500	1200	600	1000	300	600	1500	394	771	582	280	309

Scen.	S1	S2	S3	S4	S5
M_{H^\pm}	700	900	1000	1200	1400
m_{h_1}	123.487	123.509	123.517	123.529	123.539
m_{h_2}	694.483	895.594	996.769	1195.095	1397.300
m_{h_3}	695.425	896.931	996.818	1197.407	1398.600
$m_{\tilde{\tau}_1}$	282.705	282.573	282.517	282.420	282.336
$m_{\tilde{b}_1}$	513.733	513.621	513.578	513.509	513.455
$m_{\tilde{\chi}_1^0}$	295.269	295.269	295.269	295.269	295.269
$m_{\tilde{\chi}_2^0}$	476.772	476.763	476.759	476.753	476.748
$m_{\tilde{\chi}_3^0}$	496.992	496.988	496.986	496.983	496.980
$m_{\tilde{\chi}_4^0}$	632.326	632.324	632.324	632.323	632.322
$m_{\tilde{\chi}_1^\pm}$	472.534	472.534	472.534	472.534	472.534
$m_{\tilde{\chi}_2^\pm}$	632.167	632.167	632.167	632.167	632.167

$M_{1,2,3}$ [70, 71]. It is possible, for instance, to rotate the phase φ_{M_2} away. Experimental constraints on the (combinations of) complex phases arise, in particular, from their contributions to electric dipole moments of the electron and the neutron (see Refs. [72, 73] and references therein), of the deuteron [74] and of heavy quarks [75]. While SM contributions enter only at the three-loop level, due to its complex phases the MSSM can contribute already at one-loop order. Large phases in the first two generations of sfermions can only be accommodated if these generations are assumed to be very heavy [76] or large cancellations occur [77]; see, however, the discussion in Ref. [78]. A review can be found in Ref. [79]. Accordingly (using the convention that $\varphi_{M_2} = 0$, as done in this paper), in particular, the phase φ_μ is tightly constrained [80], while the bounds on the phases of the third generation trilinear couplings are much weaker. Setting $\varphi_\mu = 0$ and $\varphi_{A_f} = 0$ leaves us with φ_{M_1} as the only complex valued parameter.

Since now the complex gaugino mass parameter M_1 can appear in the couplings, contributions from absorptive parts of self-energy type corrections on external legs can arise.

The corresponding formulas for an inclusion of these absorptive contributions via finite wave function correction factors can be found in Refs. [44, 45].

We begin the numerical analysis with partial decay widths of H^\pm evaluated as a function of M_{H^\pm} , starting at $M_{H^\pm} = 600$ GeV up to $M_{H^\pm} = 1.6$ TeV, which roughly coincides with the reach of the LHC for high-luminosity running as well as an e^+e^- collider with a center-of-mass energy up to $\sqrt{s} \sim 3$ TeV [16]. Then we turn to the h_i ($i = 1, 2, 3$) decays. Finally, it should be noted that we expect from the tree-level formulas Eqs. (6) – (8) that the decay widths increase (roughly) linearly with the corresponding Higgs boson masses.

4.3.1 H^\pm decays into charginos/neutralinos

In Figs. 4 – 11 we show the results for the processes $H^\pm \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_c^\pm$ ($n = 1, 2, 3, 4$; $c = 1, 2$) as a function of M_{H^\pm} and as a function of the relevant complex phase φ_{M_1} . These are of particular interest for LHC analyses [81, 82] (as emphasized in Sect. 1). The various visible (or hardly visible) dips/thresholds occurring for different values of M_{H^\pm} in the plots are summarized in Tab. 4, labeled TC1 to TC7.

We start with the decay $H^\pm \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^\pm$. In the left plot of Fig. 4 the first (small) dip is the threshold TC1, see Tab. 4. The second (large) dip is an effect due to the threshold TC2. The third “apparently single” dip is in reality two dips coming from the thresholds TC3 and TC4. The fourth (small) dip is the threshold TC5 and the last (large) one is the threshold TC6. The size of the corrections of the partial decay widths can be especially large very close to the production threshold⁶ from which on the considered decay mode is kinematically possible. Away from this production threshold relative corrections of $\sim +10\%$ are found.

In the right plot of Fig. 4 we show the results for the complex phase φ_{M_1} varied for $M_{H^\pm} = 1000$ GeV. The full corrections are up to $\sim +13\%$ at $\varphi_{M_1} = 180^\circ$. At $\varphi_{M_1} = 90^\circ$ the H^+ (H^-) full corrections reach $\sim +12\%$ ($\sim +10\%$). Clearly visible is the \mathcal{CP} -asymmetry for the decays of the H^+ and H^- , which can reach the level of several per-cent.

In Fig. 5 we show the results for $H^\pm \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^\pm$. The tree-level decay width $\Gamma(H^\pm \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^\pm)$ is accidently very small for the parameter set chosen, see Tab. 3. Because of this smallness, the relative size of the one-loop correction becomes larger than the tree-level result, and can even turn the decay width, $\propto |\mathcal{M}_{\text{tree}}|^2 + 2 \text{Re}\{\mathcal{M}_{\text{tree}}^* \mathcal{M}_{1\text{-loop}}\}$, negative. Therefore, in this case we added $|\mathcal{M}_{1\text{-loop}}|^2$ to the full one-loop result to obtain a positive decay width. In the left plot the first (large) spike is the threshold TC2, see Tab. 4 enhanced through the two-loop contribution $|\mathcal{M}_{1\text{-loop}}|^2$ (i.e. without the explicit two-loop correction the spike would be a “usual dip”). The second “apparently single spike” (hardly visible) is (again) in reality the two thresholds TC3 and TC4. The next (apparently single) “dip” is in reality two steps (anomalous thresholds, see Ref. [83]) traced back to the C -functions $C_{0,1,2}(M_{H^\pm}^2, m_{\tilde{\chi}_2^\pm}^2, m_{\tilde{\chi}_1^0}^2, m_{\tilde{\chi}_4^0}^2, m_{\tilde{\chi}_1^\pm}^2, m_{h_1}^2)$ at $M_{H^\pm} \approx 1126$ GeV and $C_{0,1,2}(M_{H^\pm}^2, m_{\tilde{\chi}_2^\pm}^2, m_{\tilde{\chi}_1^0}^2, m_{\tilde{\chi}_2^\pm}^2, m_{\tilde{\chi}_2^0}^2, M_W^2)$ at $M_{H^\pm} \approx 1129$ GeV.⁷ Not visible (in the plot) is a

⁶ It should be noted that a calculation very close to the production threshold requires the inclusion of additional (nonrelativistic) contributions, which is beyond the scope of this paper. Consequently, very close to the production threshold our calculation (at the tree- and loop-level) does not provide a very accurate description of the decay width.

⁷ In addition both steps are contorted through the higher order contributions $|\mathcal{M}_{1\text{-loop}}|^2$.

spike, which is the threshold TC5. The last spike is the threshold TC6. Relative corrections of $\sim -90\%$ are found at $M_{H^\pm} = 1000$ GeV (see Tab. 3), where it should be kept in mind that the tree-level is already accidentally very small and thus loop corrections can have a relatively large impact.

In the right plot of Fig. 5 the results are shown for S3 as a function of φ_{M_1} . At $\varphi_{M_1} = 180^\circ$ the full corrections reach $\sim -55\%$, again related to the accidentally small tree-level result. At $\varphi_{M_1} = 90^\circ$ the H^+ (H^-) full corrections reach $\sim -59\%$ ($\sim -62\%$), showing a small \mathcal{CP} -asymmetry.

Next, in Fig. 6 the results for $H^\pm \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^\pm$ are displayed. In the left plot the results are shown as a function of M_{H^\pm} . The four visible dips here are exactly the same as in Fig. 4 (described above), beginning at $M_{H^\pm} = 976$ GeV. Relative corrections of $\sim +33\%$ ($\sim +21\%$) are found at $M_{H^\pm} = 1000$ GeV ($M_{H^\pm} = 1400$ GeV), see Tab. 3.

In the right plot the results are displayed as a function of φ_{M_1} in S3, i.e. for $M_{H^\pm} = 1000$ GeV. One can see that the size of the tree-level as well as the corrections to the partial decay width vary substantially with the complex phase φ_{M_1} . For all φ_{M_1} the full corrections lie between $+29\%$ and $+70\%$.⁸ At $\varphi_{M_1} = 90^\circ$ the H^+ (H^-) full one-loop corrections reach $\sim +29\%$ ($\sim +48\%$), i.e. the \mathcal{CP} -asymmetry is rather large with $\sim \pm 19\%$.

The decay $H^\pm \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^\pm$ is shown in Fig. 7. In the left plot the results are shown as a function of M_{H^\pm} . The first (hardly visible) dip is (again) the threshold TC5, see Tab. 4 and the second (large) one is the threshold TC6. The decay width turns out to be relatively large at $\mathcal{O}(1$ GeV). Relative corrections of $\sim +6\%$ ($\sim +4\%$) are found at $M_{H^\pm} = 1200$ GeV ($M_{H^\pm} = 1400$ GeV), see Tab. 3.

In the right plot of Fig. 7 the results are displayed as a function of φ_{M_1} in S4, i.e. for $M_{H^\pm} = 1200$ GeV. The full corrections at $\varphi_{M_1} = 180^\circ$ reach $\sim +7\%$. On the other hand it can be seen that the variation with φ_{M_1} and the \mathcal{CP} -asymmetries ($\sim \pm 0.1\%$) are rather small.

Next, in Fig. 8 the results for $H^\pm \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_1^\pm$ are displayed. In the left plot the results are shown as a function of M_{H^\pm} . Here the four visible dips are the same as in Fig. 4, beginning at $M_{H^\pm} = 976$ GeV. Relative corrections of $\sim -18\%$ ($\sim -10\%$) are found at $M_{H^\pm} = 1000$ GeV ($M_{H^\pm} = 1400$ GeV), see Tab. 3.

In the right plot the results are displayed as a function of φ_{M_1} in S3. One can see that again the tree-level results as well as the size of the corrections to the partial decay width vary substantially with the complex phase φ_{M_1} . The full corrections can reach $\sim -24\%$ and the \mathcal{CP} -asymmetry is found to be small at the level of $\sim \pm 2\%$.

In Fig. 9 we show the results for $H^\pm \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_2^\pm$. In the left plot (with M_{H^\pm} varied) the dip is (again) the threshold TC6, see Tab. 4. The decay width is found to be of the same order as for $H^\pm \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^\pm$. One-loop corrections of $\sim +5\%$ ($\sim +4\%$) are found at $M_{H^\pm} = 1200$ GeV in S4 ($M_{H^\pm} = 1400$ GeV in S5), see Tab. 3.

In the right plot of Fig. 9 the results are shown for S4 as a function of φ_{M_1} . At $\varphi_{M_1} = 180^\circ$ the full corrections reach $\sim +5\%$. At $\varphi_{M_1} = 90^\circ$ the H^+ (H^-) full corrections reach $\sim +6\%$ ($\sim +5\%$), i.e. the \mathcal{CP} -asymmetries are at the level of $\sim \pm 1\%$.

⁸ It should be noted that the loop corrections can reach $+70\%$ of the tree results because at $\varphi_{M_1} = 180^\circ$ the tree-level decay width is accidentally small, see the right plot of Fig. 6.

Table 4: Thresholds in charged Higgs boson decays.

TC1:	$M_{H^\pm} \approx 907 \text{ GeV}$	$m_{\tilde{t}_1} + m_{\tilde{b}_1} = M_{H^\pm}$
TC2:	$M_{H^\pm} = 976 \text{ GeV}$	$m_{\tilde{t}_1} + m_{\tilde{b}_2} = M_{H^\pm}$
TC3:	$M_{H^\pm} \approx 1105 \text{ GeV}$	$m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_4^0} = M_{H^\pm}$
TC4:	$M_{H^\pm} \approx 1108 \text{ GeV}$	$m_{\tilde{\chi}_2^\pm} + m_{\tilde{\chi}_2^0} = M_{H^\pm}$
TC5:	$M_{H^\pm} \approx 1135 \text{ GeV}$	$m_{\tilde{\chi}_2^\pm} + m_{\tilde{\chi}_3^0} = M_{H^\pm}$
TC6:	$M_{H^\pm} \approx 1284 \text{ GeV}$	$m_{\tilde{t}_2} + m_{\tilde{b}_1} = M_{H^\pm}$
TC7:	$M_{H^\pm} \approx 1353 \text{ GeV}$	$m_{\tilde{t}_2} + m_{\tilde{b}_2} = M_{H^\pm}$

We finish the charged Higgs-boson analysis with the decays involving the heaviest neutralino in Figs. 10 and 11, showing the results for $\Gamma(H^\pm \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_1^\pm)$ and $\Gamma(H^\pm \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_2^\pm)$, respectively.

In the left plot of Fig. 10 the first dip (not visible in the plot) is the threshold TC4, see Tab. 4. The second (small) dip is (again) the threshold TC5 and the third (large) dip is the threshold TC6. The first step (not visible in the plot) at $M_{H^\pm} \approx 1136 \text{ GeV}$ is the anomalous threshold of the C -functions $C_{0,1,2}(M_{H^\pm}^2, m_{\tilde{\chi}_1^\pm}^2, m_{\tilde{\chi}_4^0}^2, m_{\tilde{\chi}_3^0}^2, m_{\tilde{\chi}_2^\pm}^2, M_Z^2)$. The second anomalous threshold at $M_{H^\pm} \approx 1340 \text{ GeV}$ is caused by $C_{0,1,2}(m_{\tilde{\chi}_4^0}^2, M_{H^\pm}^2, m_{\tilde{\chi}_1^\pm}^2, m_b^2, m_{\tilde{b}_1}^2, m_{\tilde{t}_2}^2)$. The last dip (also not visible) is the threshold TC7. The decay width is again found at $\mathcal{O}(1 \text{ GeV})$ with relative corrections of $\sim +6\%$ in S4 (see Tab. 3).

In the right plot of Fig. 10 we show the complex phase φ_{M_1} varied at $M_{H^\pm} = 1200 \text{ GeV}$. The full corrections are up to $\sim +6\%$ at $\varphi_{M_1} = 180^\circ$. Here the asymmetries are extremely small and hardly visible.

Finally, we discuss the decay $H^\pm \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_2^\pm$ in Fig. 11. The overall size of this decay width (with real phases) is (accidentally) very small around $1 \times 10^{-3} \text{ GeV}$. Consequently, the loop corrections, can be larger than the tree-level result. In the left plot the results are shown as a function of M_{H^\pm} . The (small) dip is the threshold TC6, see Tab. 4. Relative corrections of $\sim +56\%$ are found at $M_{H^\pm} = 1400 \text{ GeV}$ (see Tab. 3).

In the right plot the results are displayed as a function of φ_{M_1} in S5. One can see that the size of the corrections to the partial decay width vary substantially with the complex phase φ_{M_1} . For all φ_{M_1} the full corrections deviate between $+40\%$ and $+146\%$. (The latter value is reached at $\varphi_{M_1} = 180^\circ$ where the tree is extremely small $\sim 1 \times 10^{-4} \text{ GeV}$.) At $\varphi_{M_1} = 90^\circ$ the H^+ (H^-) full one-loop corrections reach $\sim +40\%$ ($\sim +103\%$), i.e. the \mathcal{CP} -asymmetries are very large with $\sim \pm 60\%$.

Overall, for the charged Higgs boson decays to a chargino/neutralino pair we observe, as expected, an increasing decay width $\propto M_{H^\pm}$.⁹ The full one-loop corrections reach a level of 10% for decay widths of $\mathcal{O}(1 \text{ GeV})$. The variation with φ_{M_1} is found largest for very small decay widths, but can reach the level of 10 – 50% for widths at or below the 1 GeV level. The \mathcal{CP} -asymmetries exceed the level of a few per-cent only for very small decay widths.

⁹ An exception are the loop corrections in the left plot of Fig. 5, because there we added $|\mathcal{M}_{1\text{-loop}}|^2$.

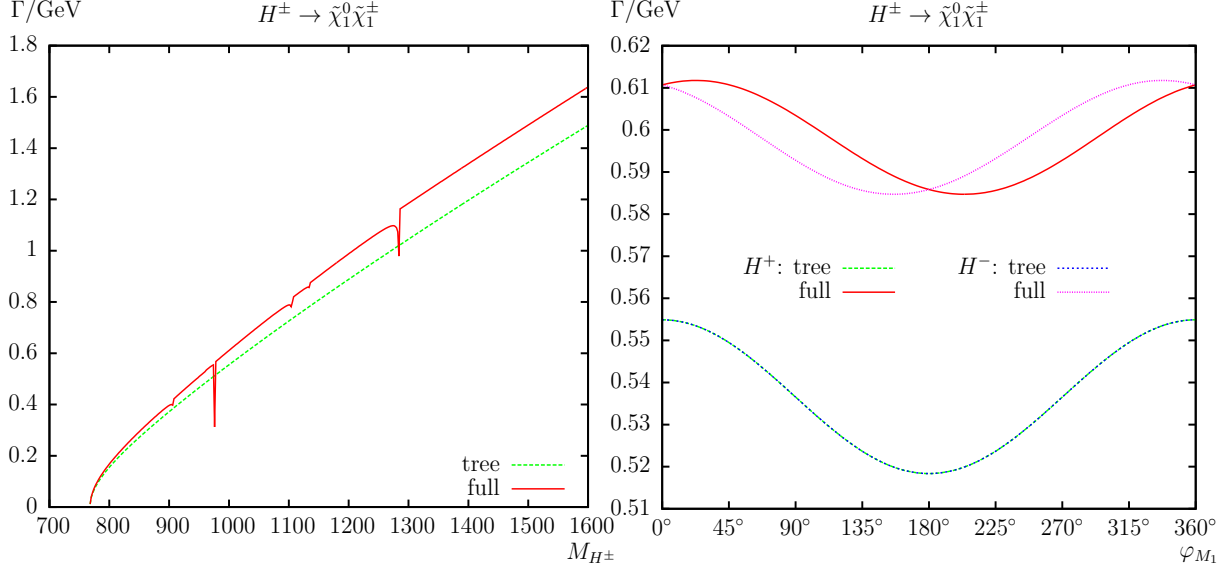


Figure 4: $\Gamma(H^\pm \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S3 (see Tab. 3).

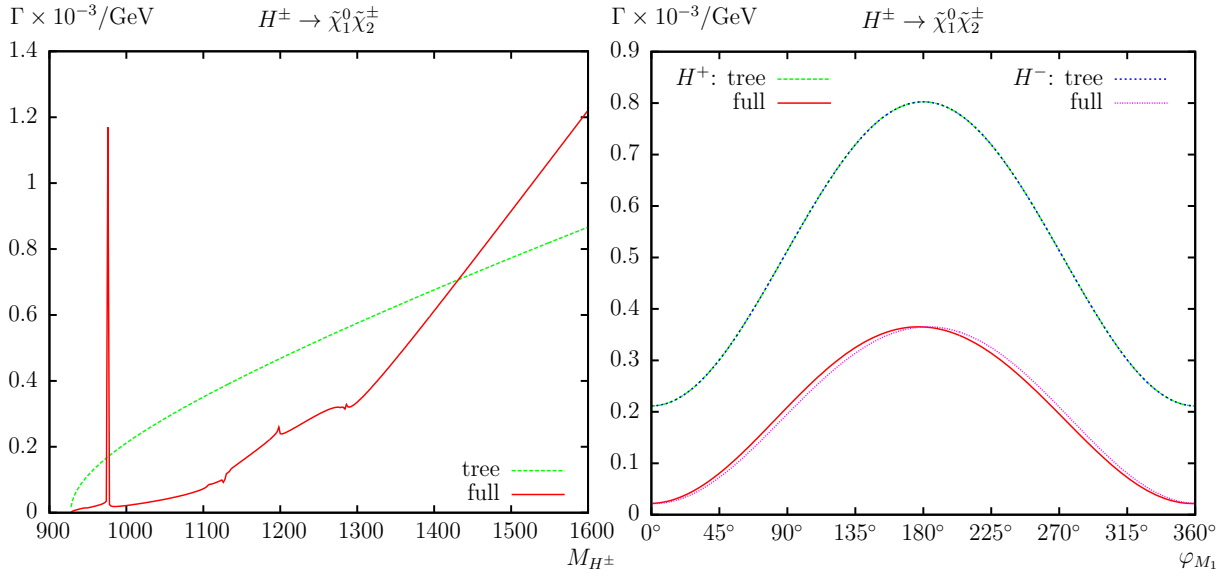


Figure 5: $\Gamma(H^\pm \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S3 (see Tab. 3).

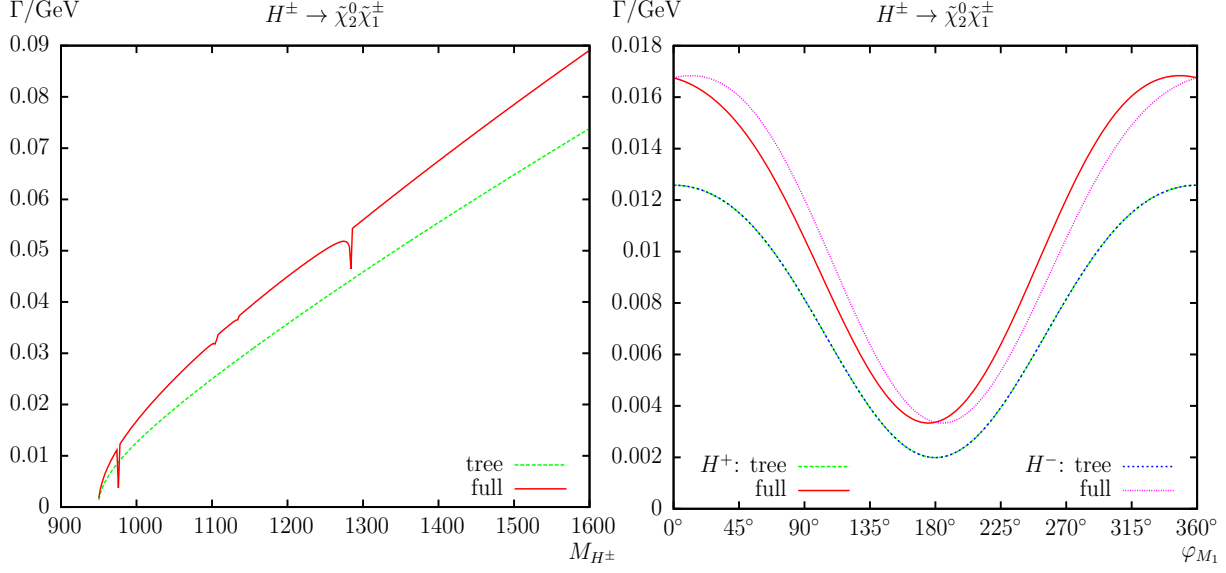


Figure 6: $\Gamma(H^\pm \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S3 (see Tab. 3).

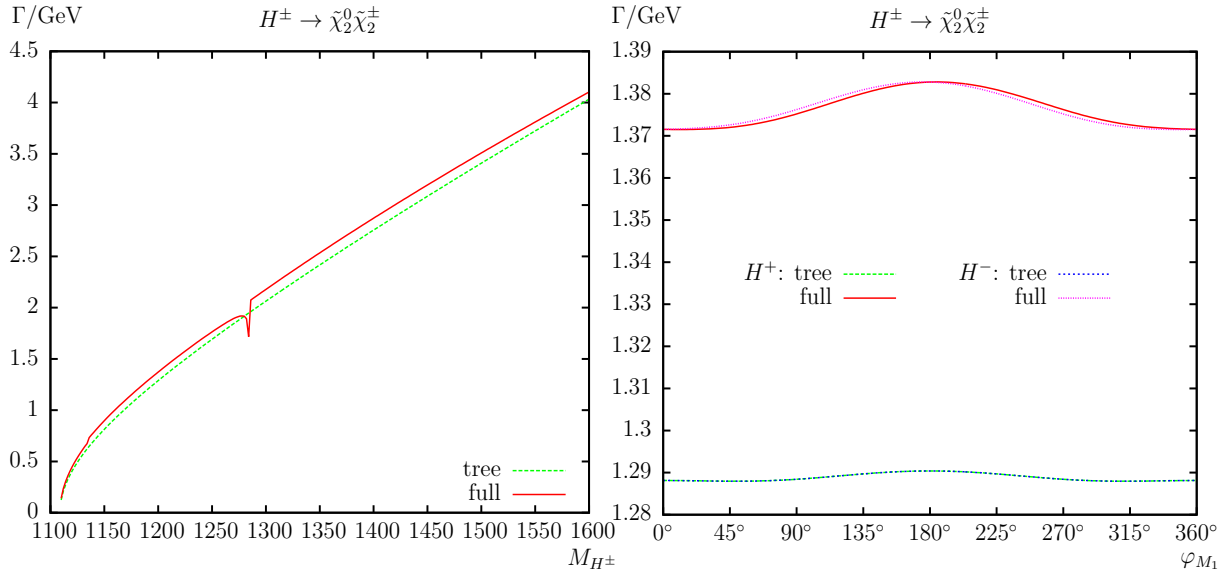


Figure 7: $\Gamma(H^\pm \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

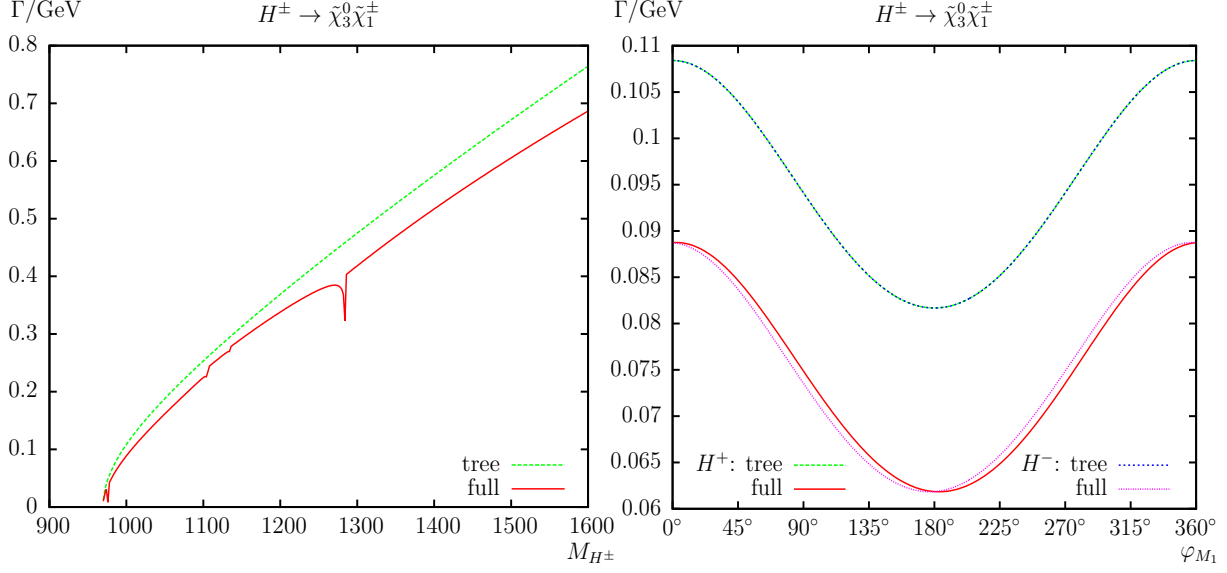


Figure 8: $\Gamma(H^\pm \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_1^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S3 (see Tab. 3).

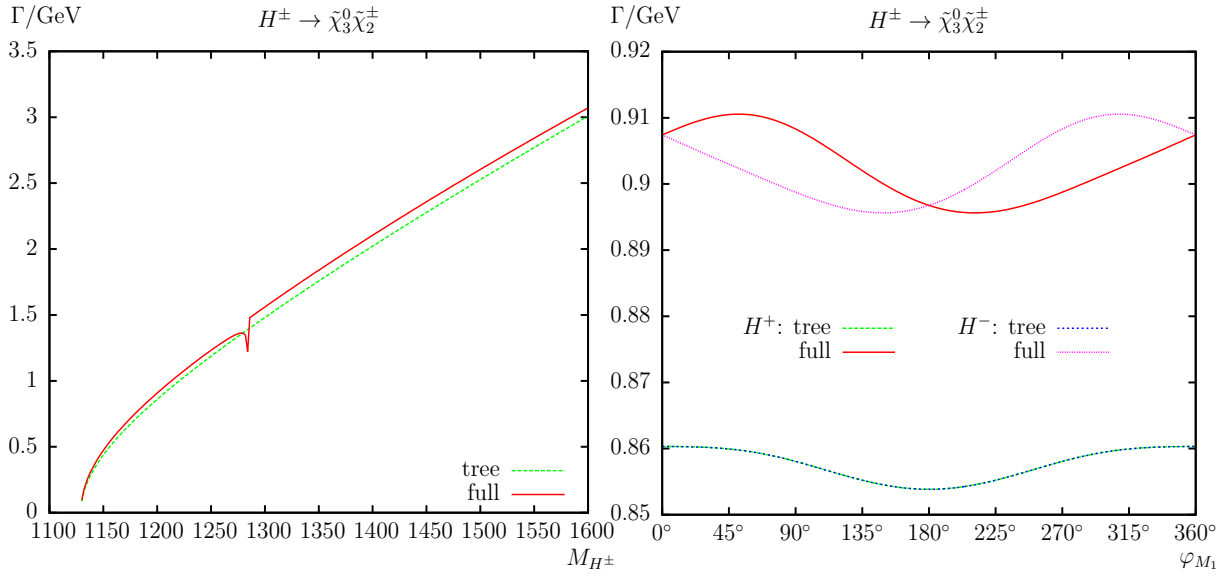


Figure 9: $\Gamma(H^\pm \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_2^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

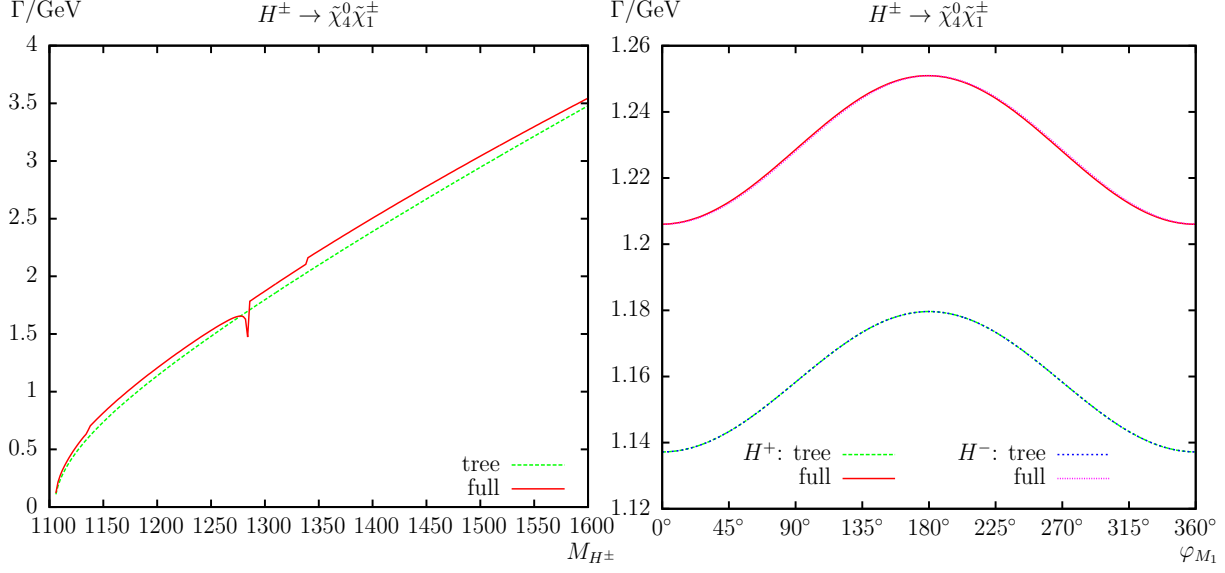


Figure 10: $\Gamma(H^\pm \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_1^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

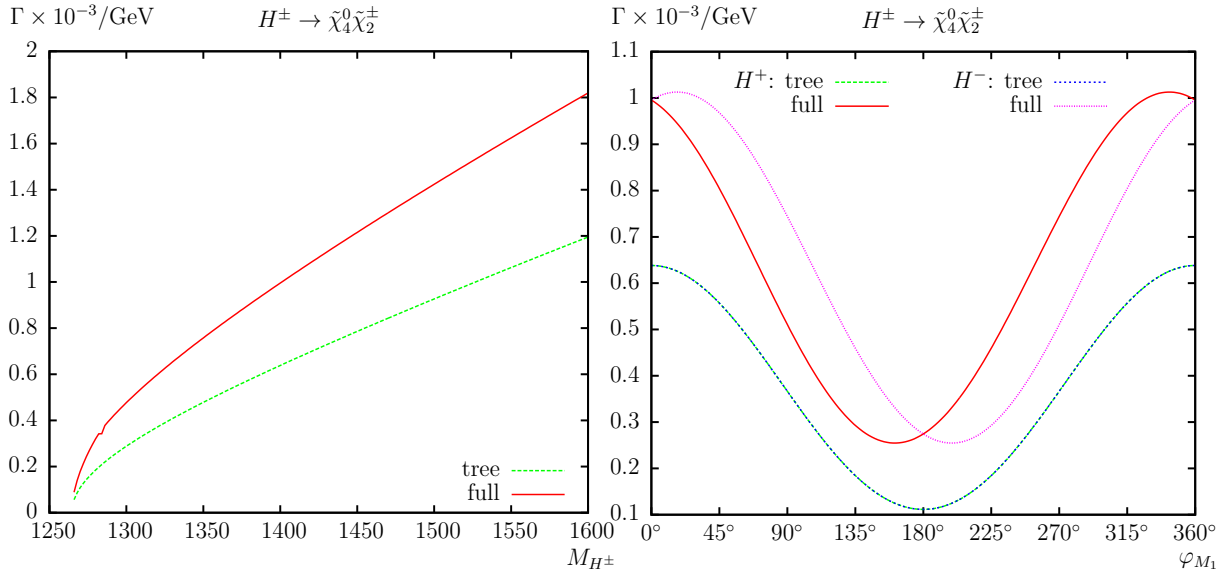


Figure 11: $\Gamma(H^\pm \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_2^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S5 (see Tab. 3).

4.3.2 h_i decays into charginos and neutralinos

We now turn to the decay modes $h_i \rightarrow \tilde{\chi}_c^- \tilde{\chi}_{c'}^+$ ($i = 2, 3; c, c' = 1, 2$) and $h_i \rightarrow \tilde{\chi}_n^0 \tilde{\chi}_{n'}^0$ ($i = 1, 2, 3; n, n' = 1, 2, 3, 4$). Results are shown in the Figs. 12 – 25.

Before discussing every figure in detail, it should be noted that there is a subtlety concerning the mixture of the h_i bosons. Depending on the input parameters, the higher-order corrections to the three neutral Higgs boson masses can vary substantially. The mass ordering $m_{h_1} < m_{h_2} < m_{h_3}$ (as performed automatically by **FeynHiggs**), even in the case of real parameters, can yield a heavy \mathcal{CP} -even Higgs mass higher *or* lower than the (heavy) \mathcal{CP} -odd Higgs mass. Such a transition in the mass ordering (or “mass crossing”) is accompanied by an abrupt change in the Higgs mixing matrix $\hat{\mathbf{Z}}$.¹⁰ For our input parameters (see Tab. 3) there are two (possible) crossings. The first (called “MC1” below) appears at $M_{H^\pm} \approx 1006$ GeV. Before the crossing we find $h_2 \sim H$ ($h_3 \sim A$), whereas after the crossing it changes to $h_2 \sim A$ ($h_3 \sim H$). The second crossing (called “MC2”) is found at $M_{H^\pm} \approx 1532$ GeV, i.e. the changing of the mixture from $h_2 \sim A$ ($h_3 \sim H$) to $h_2 \sim H$ ($h_3 \sim A$). Very close to the mass crossings the $\hat{\mathbf{Z}}$ matrix can yield small numerical instabilities. As an example, for $1532 \text{ GeV} \lesssim M_{H^\pm} \lesssim 1536 \text{ GeV}$ the $\hat{\mathbf{Z}}$ matrix causes structures appearing similar to “usual” dips from thresholds (see also the discussion in Ref. [42]). All the dips/thresholds (some are hardly visible) appearing in the figures below are listed in Tab. 5, labeled as TN1 to TN13.

h_i decays into charginos

In this subsection we analyze the decays of the heavy neutral Higgs bosons into charginos. We start with the decay $h_i \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_1^\pm$ ($i = 2, 3$) as shown in Fig. 12. The left plot shows the results as a function of M_{H^\pm} , whereas in the right plot we present the decay widths as a function of φ_{M_1} in S4. We show separately the results for the h_2 and h_3 decay widths. In the left plot of Fig. 12 the first “apparently single” dip in the h_2 decay (upper lines) is in reality coming from the thresholds TN6 and TN7, see Tab. 5. The second (large) dip is the threshold TN10. The last “apparently single” dip is in reality coming from the thresholds TN11 and TN12. The “step” (anomalous threshold) at $M_{H^\pm} \approx 1310$ GeV could be traced back to the C -functions $C_{0,1,2}(m_{\tilde{\chi}_1^\pm}^2, m_{h_2}^2, m_{\tilde{\chi}_1^\pm}^2, m_b^2, m_{t_s}^2, m_{t_{s'}}^2)$ with $s \neq s'$. Away from the production threshold relative corrections of $\sim -3\%$ are found in S4 (see Tab. 3) for the h_2 decay. The loop corrections increase with increasing M_{H^\pm} and reach $\sim -10\%$ in S5. In case of the h_3 decay the dips are the same as for h_2 and the relative corrections are only $\sim -2\%$ in S4 (see Tab. 3). The two mass crossings MC1 and MC2 are clearly visible at $M_{H^\pm} \approx 1006$ GeV and $M_{H^\pm} \approx 1532$ GeV as described above, where h_2 and h_3 change their role. Between MC1 and MC2 we find $\Gamma(h_2 \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_1^\pm) > \Gamma(h_3 \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_1^\pm)$, outside it is vice versa, as can be clearly observed in the left plot of Fig. 12. The suppression of the \mathcal{CP} -even decay (lower lines) vs. the \mathcal{CP} -odd decay (upper lines) is clearly visible, where at threshold the behavior follows Eqs. (9), (10). After the threshold the decays grow roughly linear with the Higgs boson masses.

We now turn to the phase dependence of the decay width shown in S4, i.e. for $M_{H^\pm} =$

¹⁰In our case the Z -factor matrix is given by $\hat{Z}_{ij} \equiv \mathbf{ZHiggs}[i, j]$, see Ref. [45] (and Ref. [26]), which contributes at tree-level. Furthermore $\hat{\mathbf{Z}}$ is calculated by **FeynHiggs** which uses $m_b(m_b)$ and tree-level sfermion masses instead of the shifted masses, causing a slight displacement in the threshold position.

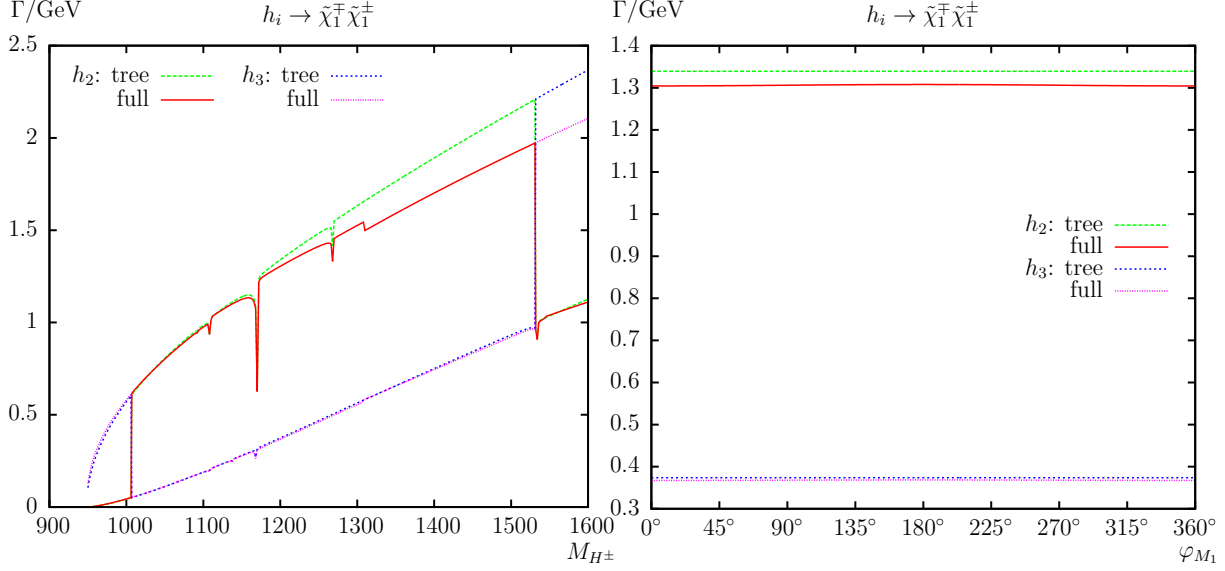


Figure 12: $\Gamma(h_i \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_1^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

1200 GeV, where the right plot in Fig. 12 shows the dependence of $\Gamma(h_i \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_1^\pm)$ on φ_{M_1} . Since M_1 does not appear in the chargino mass matrix, the effects of varying its phase enter only via loop corrections and are extremely small. The relative corrections in S4 are the same as in the left plot.

The results for $\Gamma(h_i \rightarrow \tilde{\chi}_2^\mp \tilde{\chi}_2^\pm)$, as shown in the left plot of Fig. 13 are smaller by roughly a factor of 2 w.r.t. $\Gamma(h_i \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_1^\pm)$, largely related to the kinematic suppression. At $M_{H^\pm} = 1400$ GeV the full one-loop corrections to the h_2 decay reach $\sim +4\%$. For the decay of the h_3 at $M_{H^\pm} = 1400$ GeV we find full corrections at the level of less than $+1\%$. As in the upper left plot one can observe the MC2 with an “interchange” of h_2 and h_3 . The same suppression of the \mathcal{CP} -even vs. the \mathcal{CP} -odd decay, as in Fig. 14 is clearly visible.

In the right plot of Fig. 13 we show the complex phase φ_{M_1} varied at $M_{H^\pm} = 1400$ GeV. The variation with φ_{M_1} is extremely small (for the same reasons as explained above), therefore the full relative corrections in S5 are the same as in the left plot, see above.

The results for the “mixed” decay, $\Gamma(h_i \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_2^\pm)$, are shown in Fig. 14, where in the left (right) plot we show the dependence on M_{H^\pm} (φ_{M_1}). In the left plot the first dip in the h_2 decay (lower lines) is the threshold TN10, see Tab. 5. The remaining dip (at $M_{H^\pm} \approx 1268$ GeV) is caused by the two thresholds TN11 and TN12. At $M_{H^\pm} = 1200$ GeV the full one-loop corrections to the h_2 decay reach $\sim +20\%$. Now we turn to the corresponding h_3 decay. The first dip (hardly visible in the upper lines) is the threshold TN8. The second dip is the same as for the h_2 decay, see above. For the decay of h_3 at $M_{H^\pm} = 1200$ GeV we find full corrections at the level of $+10\%$. As in Fig. 12 one can observe the MC2 with an “interchange” of h_2 and h_3 .

Table 5: Thresholds in neutral Higgs boson decays.

TN1:	$M_{H^\pm} \approx 805 \text{ GeV}$	$m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_3^0} = m_{h_2} \approx 799 \text{ GeV}$
TN2:	$M_{H^\pm} \approx 948 \text{ GeV}$	$m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_1^\pm} = m_{h_3} \approx 945 \text{ GeV}$
TN3:	$M_{H^\pm} \approx 954 \text{ GeV}$	$m_{\tilde{\chi}_2^0} + m_{\tilde{\chi}_2^0} = m_{h_3} \approx 951 \text{ GeV}$
TN4:	$M_{H^\pm} \approx 1092 \text{ GeV}$	$m_{\tilde{b}_1} + m_{\tilde{b}_2} = m_{h_2} \approx 1086 \text{ GeV}$
TN5:	$M_{H^\pm} \approx 1107 \text{ GeV}$	$m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_2^\pm} = m_{h_3} \approx 1105 \text{ GeV}$
TN6:	$M_{H^\pm} \approx 1108 \text{ GeV}$	$m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_2^\pm} = m_{h_2} \approx 1105 \text{ GeV}$
TN7:	$M_{H^\pm} \approx 1112 \text{ GeV}$	$m_{\tilde{\chi}_2^0} + m_{\tilde{\chi}_4^0} = m_{h_2} \approx 1108 \text{ GeV}$
TN8:	$M_{H^\pm} \approx 1138 \text{ GeV}$	$m_{\tilde{\chi}_3^0} + m_{\tilde{\chi}_4^0} = m_{h_3} \approx 1135 \text{ GeV}$
TN9:	$M_{H^\pm} \approx 1168 \text{ GeV}$	$m_{\tilde{t}_1} + m_{\tilde{t}_2} = m_{h_3} = 1165 \text{ GeV}$
TN10:	$M_{H^\pm} \approx 1171 \text{ GeV}$	$m_{\tilde{t}_1} + m_{\tilde{t}_2} = m_{h_2} = 1165 \text{ GeV}$
TN11:	$M_{H^\pm} \approx 1268 \text{ GeV}$	$m_{\tilde{\chi}_2^\pm} + m_{\tilde{\chi}_2^\pm} = m_{h_2} \approx 1264 \text{ GeV}$
TN12:	$M_{H^\pm} \approx 1268 \text{ GeV}$	$m_{\tilde{\chi}_4^0} + m_{\tilde{\chi}_4^0} = m_{h_2} \approx 1265 \text{ GeV}$
TN13:	$M_{H^\pm} \approx 1545 \text{ GeV}$	$m_{\tilde{t}_2} + m_{\tilde{t}_2} = m_{h_2} = 1542 \text{ GeV}$

In the right plot of Fig. 14 one can see that the variation with φ_{M_1} is again very small with tiny \mathcal{CP} -asymmetries and the same corrections as in the left plot (for the same reasons as explained above).

Overall, for the neutral Higgs decays to a chargino pair we observe, again as expected, an increasing decay width $\propto M_{H^\pm}$, as $m_{h_{2,3}}$ increase nearly linearly with our input parameter M_{H^\pm} . The full one-loop corrections reach a level of 10% for decay widths being of $\mathcal{O}(1 \text{ GeV})$, and they can reach up to 20% in the “mixed” decay mode. The variation with φ_{M_1} is found to be negligible, as expected, since M_1 enters only via the loop corrections.

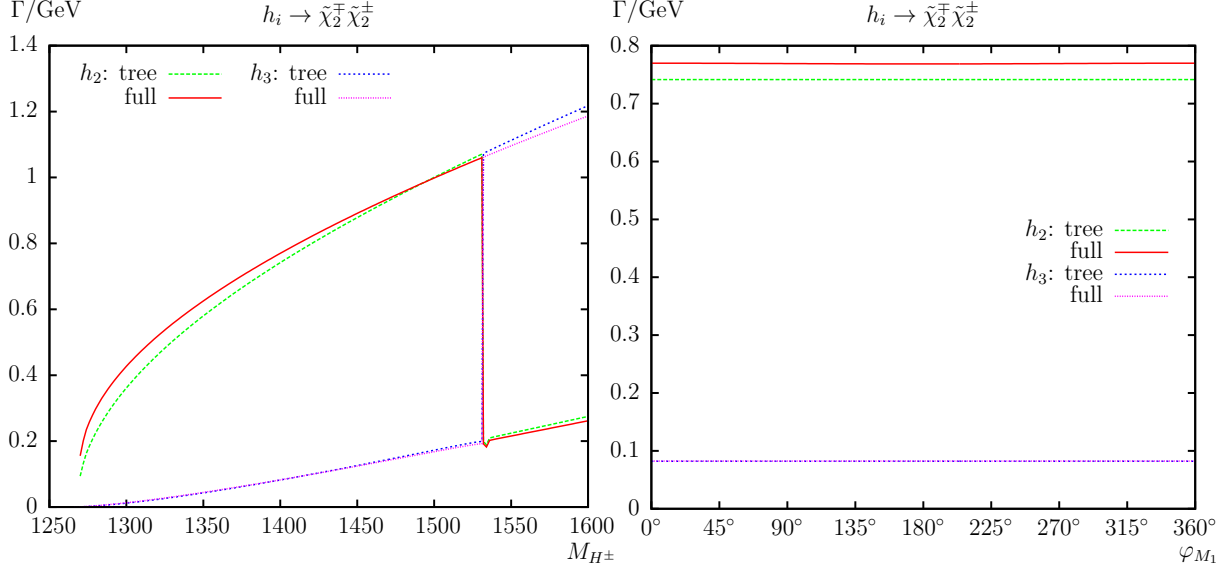


Figure 13: $\Gamma(h_i \rightarrow \tilde{\chi}_2^\mp \tilde{\chi}_2^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S5 (see Tab. 3).

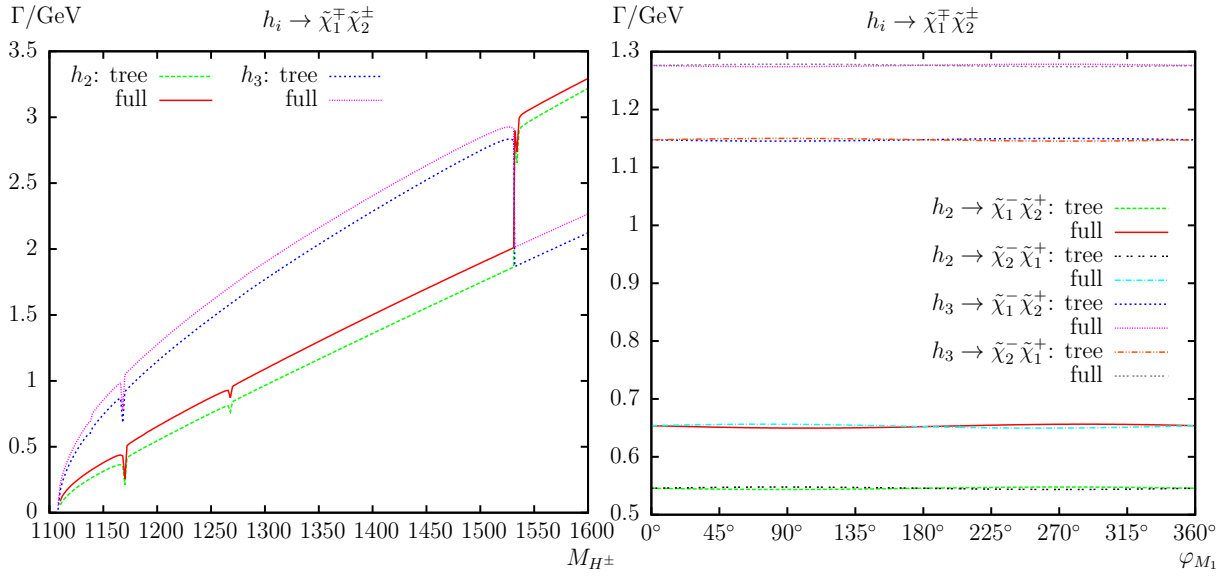


Figure 14: $\Gamma(h_i \rightarrow \tilde{\chi}_1^\mp \tilde{\chi}_2^\pm)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

h_i decays into neutralinos

Next we consider h_i decays into neutralinos with equal indices. First, we present the decay $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ in Fig. 15. Bounds on $m_{\tilde{\chi}_1^0}$ often assume an underlying GUT based on a simple Lie group, leading to $M_1 = 5/3(s_w/c_w)^2 M_2$. If the latter assumption is dropped, hardly any direct bound on $m_{\tilde{\chi}_1^0}$ can be placed [36]. Therefore, we also treat M_1 as an independent parameter. The decay $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ constitutes an invisible decay of the light Higgs boson, where bounds based on a combination of LHC and Tevatron data yield an upper bound on an invisible branching ratio of $\sim 40\%$ [84]. Since the $\tilde{\chi}_1^0$ constitutes a perfect Dark Matter candidate in the MSSM [35], in order to determine the Dark Matter properties a precision measurement of this process at the LHC or a future e^+e^- collider will be necessary. Consequently, a precise prediction of $\Gamma(h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0)$ is of particular interest.

In the upper left plot of Fig. 15 we show the results for M_{H^\pm} varied in S1, but with $|M_1| = 50$ GeV as the base scenario. The full loop corrections are $\sim +25\%$ at $M_{H^\pm} = 700$ GeV in the upper left plot. φ_{M_1} is varied in the upper right plot. One can observe a strong dependence of the decay width on φ_{M_1} , which can change by a factor of 8. The largest loop corrections are found as $\sim +31\%$ for $\varphi_{M_1} = 72^\circ, 288^\circ$ and $\sim +59\%$ at $\varphi_{M_1} = 180^\circ$. In the lower left plot of Fig. 15 we show the decay width with M_1 varied. Close to $M_1 = 0$ the lightest neutralino becomes massless. For not too small values a decay width of $\sim 10^{-4}$ GeV can be observed, going to zero at the kinematic threshold. The one-loop corrections reach up to $\sim +30\%$ at $M_1 = 20$ GeV. Finally, in the lower right plot $|\mu|$ is varied, and the decay width drops down to $\sim 10^{-5}$ GeV for $\mu > 600$ GeV and with corrections between $\sim +6\%$ and $\sim +28\%$.

We now turn to the decays of the heavy neutral Higgs bosons. In Fig. 16 we present the results for the decays $h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ ($i = 2, 3$). The dependence on M_{H^\pm} is shown in the left plot, whereas the dependence on φ_{M_1} for $M_{H^\pm} = 700$ GeV is given in the right plot. We start with $\Gamma(h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0)$ in the left plot. The first dip (lower lines) in the h_2 decay is the threshold TN1, see Tab. 5. The second dip (hardly visible in the upper lines) is the threshold TN4.¹¹ The third “apparently single” dip is (again) in reality coming from the thresholds TN6 and TN7. The fourth (large) dip is the threshold TN10. The last “apparently single” dip is in reality coming from the thresholds TN11 and TN12. The full loop corrections are $\sim +11\%$ at $M_{H^\pm} = 700$ GeV. Also shown in this plot is the decay $h_3 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$. The first dip (upper lines) is in reality coming from the thresholds TN2 and TN3. The second dip (lower lines) in the h_3 decay is the threshold TN5. The third dip (lower lines) is the threshold TN8 and the last dip is the threshold TN9. The full relative corrections reach $\sim +10\%$ at $M_{H^\pm} = 700$ GeV. The suppression of the \mathcal{CP} -even decay (lower lines, going with Eq. (10) at threshold, and then roughly linear with m_{h_2}) vs. the \mathcal{CP} -odd decay (upper lines, going with Eq. (9) at threshold, and then roughly linear with m_{h_3}) is again clearly visible.

In the right plot of Fig. 16 we show the h_2 decay with the complex phase φ_{M_1} varied at $M_{H^\pm} = 700$ GeV. The variation with φ_{M_1} is found to be very large, changing the decay width

¹¹ It should be noted that this threshold enter *into the tree-level* only via the $\hat{\mathbf{Z}}$ matrix contribution. These effects propagate also into the loop corrections via $2\text{Re}\{\mathcal{M}_{\text{tree}}^* \mathcal{M}_{1\text{-loop}}\}$. Furthermore $\hat{\mathbf{Z}}$ is calculated by `FeynHiggs` which uses $m_b(m_b)$ and tree-level sfermion masses instead of the shifted masses, causing a slight displacement in the threshold position.

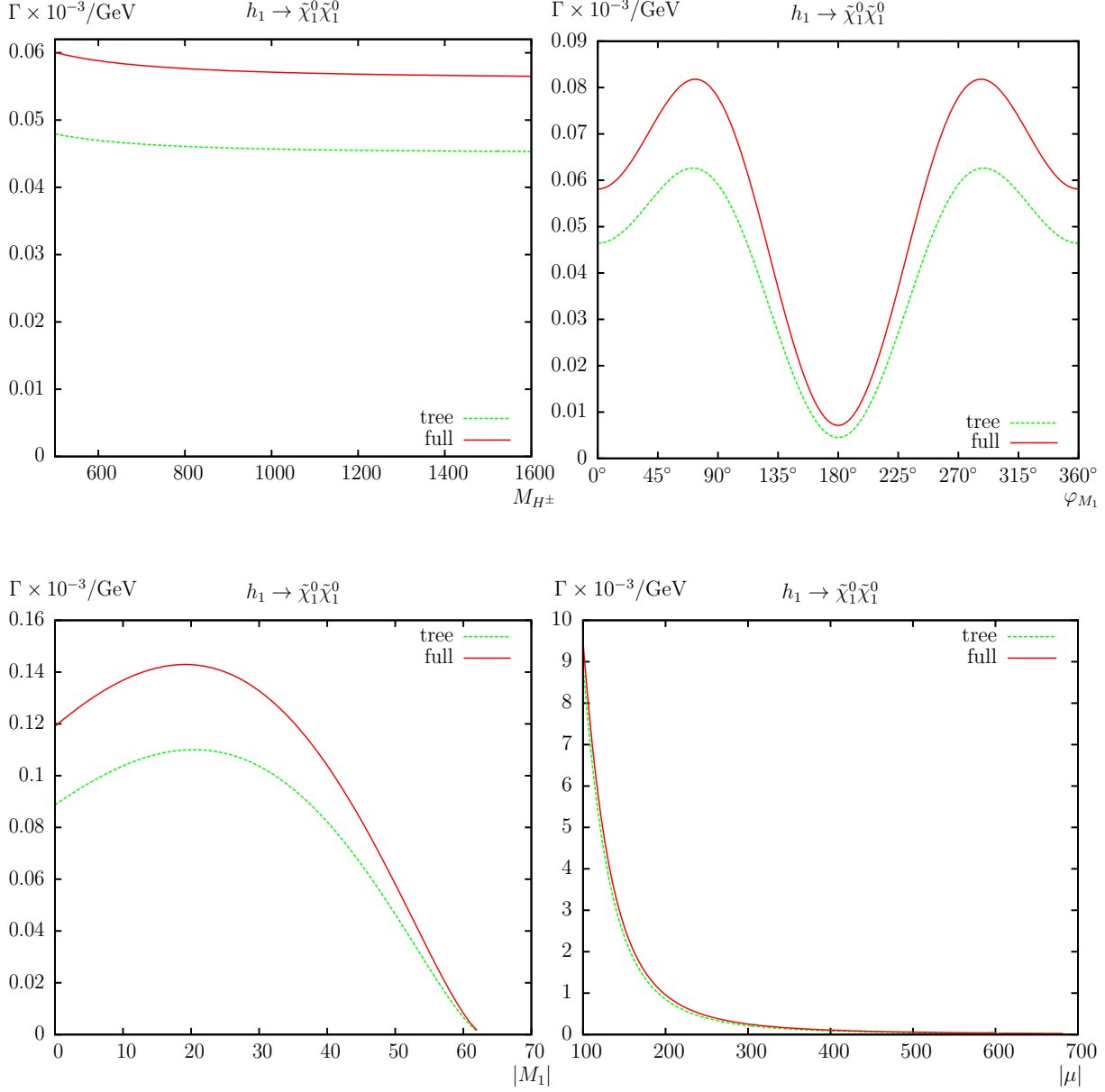


Figure 15: $\Gamma(h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0)$. Tree-level and full one-loop corrected partial decay widths are shown with parameters chosen according to S1 (see Tab. 3) but here with $|M_1| = 50$ GeV. The upper left plot shows the partial decay width with M_{H^\pm} varied. The upper right plot shows the complex phase φ_{M_1} varied. The lower left (right) plot shows M_1 ($|\mu|$) varied.

by up to a factor of 5 where the full relative corrections are up to $\sim +20\%$ at $\varphi_{M_1} = 180^\circ$ for S1. The h_3 decay with the complex phase φ_{M_1} shows also a very large variation at $M_{H^\pm} = 700$ GeV and the loop corrections reach up to $\sim +19\%$ at $\varphi_{M_1} = 180^\circ$.

In Fig. 17 we show the decays $h_i \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$, in full analogy to Fig. 16. The same behavior of h_2 and h_3 concerning MC1 and MC2, as well as the \mathcal{CP} -even and \mathcal{CP} -odd decay can be

observed. The dips (some are hardly visible) are the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 1092$ GeV, see above. The full relative corrections for the h_2 (h_3) decay are $\sim -18\%$ ($\sim +10\%$) at $M_{H^\pm} = 1200$ GeV, i.e. S4.

In the right plot of Fig. 17 we show the variation of $\Gamma(h_2 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0)$ with φ_{M_1} at $M_{H^\pm} = 1200$ GeV. Here the loop corrections can vary between $\sim -18\%$ for $\varphi_{M_1} = 0^\circ, 360^\circ$ and $\sim -13\%$ at $\varphi_{M_1} = 180^\circ$. The h_3 decay with φ_{M_1} varied reach $\sim +11\%$ for $\varphi_{M_1} \sim 90^\circ, 270^\circ$ in S4.

Next, in Fig. 18 we present the decays $h_i \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_3^0$, in full analogy to Fig. 16. The same behavior of h_2 and h_3 concerning MC1 and MC2 and the \mathcal{CP} -even/-odd decay can be observed. The dips (some are hardly visible) are again the same as described in Fig. 16 beginning at $M_{H^\pm} \approx 1092$ GeV, see above. The “knee” at $M_{H^\pm} \approx 1545$ GeV (red line) is the threshold TN13 (see Tab. 5) in the C -functions $C_{0,1,2}(m_{\tilde{\chi}_3^0}^2, m_{h_2}^2, m_{\tilde{\chi}_3^0}^2, m_t^2, m_{t_2}^2, m_{t_2}^2)$. The full one-loop corrections for the h_2 decay are $\sim +172\%$ at $M_{H^\pm} = 1200$ GeV. This strange behavior is a numerical effect caused by an interplay of anomalous thresholds in $C_{0,1,2}(m_{\tilde{\chi}_3^0}^2, m_{h_2}^2, m_{\tilde{\chi}_3^0}^2, m_t^2, m_{t_s}^2, m_{t_{s'}}^2)$ ($s \neq s'$) with the effects induced by the Higgs mixing matrix $\hat{\mathbf{Z}}$. This effect is absent in the decay of the h_3 , where we find the full relative corrections at the level of $\sim +25\%$ for $M_{H^\pm} = 1200$ GeV.

In the right plot of Fig. 18 we show the variation of $\Gamma(h_2 \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_3^0)$ with φ_{M_1} at $M_{H^\pm} = 1200$ GeV (i.e. at an “extreme” point for the h_2 decay). Here (for the same reasons as in the left plot) the loop corrections reach $\sim +111\%$ at $\varphi_{M_1} = 180^\circ$. Also in the right plot of Fig. 18 we show $\Gamma(h_3 \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_3^0)$ with φ_{M_1} varied in S4. Here the loop corrections can reach $\sim +26\%$ at $\varphi_{M_1} = 90^\circ, 270^\circ$.

In Fig. 19 we present the decays $h_i \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_4^0$, again in full analogy to Fig. 16. The same behavior of h_2 and h_3 concerning MC2 and the \mathcal{CP} -even/-odd decay can be observed. The full relative corrections for the h_2 decay are $\sim +4\%$ at $M_{H^\pm} = 1400$ GeV, i.e. S5, while the h_3 decay shows relative corrections less than $+1\%$ at $M_{H^\pm} = 1400$ GeV.

In the right plot of Fig. 19 we show $\Gamma(h_{2,3} \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_4^0)$ at $M_{H^\pm} = 1400$ GeV. For both decays the variation of φ_{M_1} is very small, i.e. the loop corrections reach the same values (in S5) as in the left plot.

We now turn to the neutral Higgs decays to neutralinos with different indices. In this case, contrary to the decay into identical charginos, the \mathcal{CP} -asymmetries are also zero, due to the Majorana nature of the neutralinos.

In Fig. 20 we present the decay $h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$. In the left plot we show the results as a function of M_{H^\pm} . The dips are (again) the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 948$ GeV, see above. The full relative corrections for the h_2 decay are $\sim +9\%$ at $M_{H^\pm} = 900$ GeV (i.e. S2). The full one-loop corrections for the h_3 decay at $M_{H^\pm} = 900$ GeV reach $+7\%$.

In the right plot of Fig. 20 we show the h_i decay with the complex phase φ_{M_1} varied at $M_{H^\pm} = 900$ GeV. The variation with φ_{M_1} is found to be very large and the loop corrections vary between $\sim +9\%$ for $\varphi_{M_1} \sim 0^\circ$ and $\sim +6\%$ at $\varphi_{M_1} = 90^\circ$. We also show the h_3 decay in the right plot of Fig. 20 with φ_{M_1} varied at $M_{H^\pm} = 900$ GeV. The variation with φ_{M_1} is yet larger than in the h_2 case. The full corrections can reach $\sim +10\%$ at $\varphi_{M_1} = 180^\circ$. The general behavior can be understood as follows. For $\varphi_{M_1} = 0^\circ$ one finds

$\mathcal{CP}(h_2\tilde{\chi}_1^0\tilde{\chi}_2^0) = -\mathcal{CP}(h_3\tilde{\chi}_1^0\tilde{\chi}_2^0) = +1$, leading to the above discussed suppression of the h_2 decay, see Eqs. (9), (10). Going to $\varphi_{M_1} = 180^\circ$ changes the \mathcal{CP} -nature of the $\tilde{\chi}_1^0$, leading to $\mathcal{CP}(h_2\tilde{\chi}_1^0\tilde{\chi}_2^0) = -\mathcal{CP}(h_3\tilde{\chi}_1^0\tilde{\chi}_2^0) = -1$ and the corresponding suppression of the h_3 decay.

In Fig. 21 we present the decay $h_i \rightarrow \tilde{\chi}_1^0\tilde{\chi}_3^0$. It should be noted that the decay $h_i \rightarrow \tilde{\chi}_1^0\tilde{\chi}_3^0$ looks quite similar to Fig. 20 but with an interchange of h_2 with h_3 . In the left plot we show the results as a function of M_{H^\pm} . The dips are (again) the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 948$ GeV, see above. The full one-loop corrections for the h_2 decay reach $\sim +7\%$ at $M_{H^\pm} = 900$ GeV (i.e. S2). The relative corrections for the h_3 decay at $M_{H^\pm} = 900$ GeV are $+14\%$. In comparison with Fig. 20 one can observe an “inversion” of the relative size of the decays widths of the h_2 and the h_3 (green/red lines vs. blue/purple lines). This “inversion” is due to the fact that $\mathcal{CP}(\tilde{\chi}_1^0\tilde{\chi}_2^0) = -\mathcal{CP}(\tilde{\chi}_1^0\tilde{\chi}_3^0)$.

In the right plot of Fig. 21 we show the h_i decay with the complex phase φ_{M_1} varied at $M_{H^\pm} = 900$ GeV. This variation is (again) found to be very large, as can be seen in the right plot. The loop corrections for the h_2 decay can reach $\sim +9\%$ at $\varphi_{M_1} = 180^\circ$. In the right plot of Fig. 21 we show also the h_3 decay with φ_{M_1} varied at $M_{H^\pm} = 900$ GeV. There the loop corrections vary between $\sim +14\%$ for $\varphi_{M_1} \sim 0^\circ$ and $\sim +7\%$ at $\varphi_{M_1} = 90^\circ$. Again the “inversion” (as in the left plot) can be observed.

In Fig. 22 we present the results for $\Gamma(h_i \rightarrow \tilde{\chi}_1^0\tilde{\chi}_4^0)$ as a function of M_{H^\pm} in the left plot. The tree-level decay width is accidentally very small for the parameter set chosen, see Tab. 3. Because of this smallness, the relative size of the one-loop correction becomes larger than the tree-level, and can even turn negative. Therefore, in this case we added $|\mathcal{M}_{1\text{-loop}}|^2$ to the full one-loop result to obtain a positive decay width. The dips are (again) the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 948$ GeV, see above. The anomalous thresholds (“steps” in the red line) could be traced back to the C -functions at

$$\begin{aligned} M_{H^\pm} \approx 1020 \text{ GeV} : & \quad C_{0,1,2}(m_{h_2}^2, m_{\tilde{\chi}_4^0}^2, m_{\tilde{\chi}_1^0}^2, m_{\tilde{\chi}_3^0}^2, m_{\tilde{\chi}_3^0}^2, M_Z^2), \\ M_{H^\pm} \approx 1026 \text{ GeV} : & \quad C_{0,1,2}(m_{h_2}^2, m_{\tilde{\chi}_4^0}^2, m_{\tilde{\chi}_1^0}^2, m_{\tilde{\chi}_2^\pm}^2, m_{\tilde{\chi}_1^\pm}^2, M_W^2), \\ M_{H^\pm} \approx 1031 \text{ GeV} : & \quad C_{0,1,2}(m_{\tilde{\chi}_1^0}^2, m_{h_2}^2, m_{\tilde{\chi}_4^0}^2, m_b^2, m_{b_2}^2, m_{b_1}^2), \\ M_{H^\pm} \approx 1035 \text{ GeV} : & \quad C_{0,1,2}(m_{h_2}^2, m_{\tilde{\chi}_4^0}^2, m_{\tilde{\chi}_1^0}^2, m_{\tilde{\chi}_4^0}^2, m_{\tilde{\chi}_2^0}^2, m_{h_1}^2), \\ M_{H^\pm} \approx 1182 \text{ GeV} : & \quad C_{0,1,2}(m_{\tilde{\chi}_1^0}^2, m_{h_2}^2, m_{\tilde{\chi}_4^0}^2, m_t^2, m_{t_2}^2, m_{t_1}^2), \end{aligned}$$

(in this order). The full relative corrections for the h_2 (h_3) decay are $\sim -77\%$ ($\sim -93\%$) at $M_{H^\pm} = 1200$ GeV (i.e. S4).

In the right plot of Fig. 22 we show $\Gamma(h_i \rightarrow \tilde{\chi}_1^0\tilde{\chi}_4^0)$ with the complex phase φ_{M_1} varied at $M_{H^\pm} = 1200$ GeV. Here (for the same reasons as in the left plot) the loop corrections can be larger than the tree-level (and for consistency with the left plot we also add $|\mathcal{M}_{1\text{-loop}}|^2$ here). The loop corrections for the h_2 decay vary between $\sim -77\%$ at $\varphi_{M_1} \sim 0^\circ, 360^\circ$ and $\sim -44\%$ at $\varphi_{M_1} = 180^\circ$. The loop corrections for the h_3 decay vary between $\sim -93\%$ at $\varphi_{M_1} \sim 0^\circ, 360^\circ$ and $\sim -78\%$ at $\varphi_{M_1} = 180^\circ$.

In Fig. 23 we present the decay $h_i \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0$. In the left plot we show the results as a function of M_{H^\pm} . The dips (some are hardly visible) are again the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 1092$ GeV, see above. The full relative corrections for the h_2

decay reach up to $\sim +59\%$ at $M_{H^\pm} = 1200$ GeV (i.e. S4). The loop corrections for the h_3 decay at $M_{H^\pm} = 1200$ GeV are $\sim -14\%$.

In the right plot of Fig. 23 we show the h_i decay with the complex phase φ_{M_1} varied at $M_{H^\pm} = 1200$ GeV. For the h_2 decay the variation of φ_{M_1} is very small, i.e. the loop corrections reach (in S4) the same values ($\sim +59\%$) for all φ_{M_1} . The variation of φ_{M_1} in the h_3 decay is also small with corrections at the level of $\sim -14\%$.

In Fig. 24 we show the decay $h_i \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_4^0$. In the left plot we show the results as a function of M_{H^\pm} . The dips are (again) the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 1171$ GeV, see above. The full relative corrections for the h_2 decay reach up to $\sim +15\%$ at $M_{H^\pm} = 1200$ GeV, i.e. S4. The loop corrections for the h_3 decay at $M_{H^\pm} = 1200$ GeV are $\sim +9\%$.

In the right plot of Fig. 24 the h_i decay is shown with the complex phase φ_{M_1} varied in S4. For both decays the variation of φ_{M_1} is very small, as expected, since $\tilde{\chi}_2^0$ and $\tilde{\chi}_4^0$ are determined largely by M_2 and μ in the neutralino mass matrix (for the parameters chosen as in Tab. 3). The loop corrections for the h_2 (h_3) decay reach $\sim +16\%$ ($\sim +9\%$) at $\varphi_{M_1} = 180^\circ$.

The final decays involving neutralinos are shown in Fig. 25. The results as a function of M_{H^\pm} are given in the left plot. The dips are (again) the same as already described in Fig. 16 beginning at $M_{H^\pm} \approx 1171$ GeV, see above. The full relative corrections are only $\sim +3\%$ at $M_{H^\pm} = 1200$ GeV (i.e. S4). The full relative corrections at $M_{H^\pm} = 1200$ GeV reach $+6\%$. In comparison with Fig. 24 one can observe (again) an “inversion” of the relative size of the decays widths of the h_2 and the h_3 , due to the fact that $\mathcal{CP}(\tilde{\chi}_2^0 \tilde{\chi}_4^0) = -\mathcal{CP}(\tilde{\chi}_3^0 \tilde{\chi}_4^0)$.

In the right plot of Fig. 25 we show the h_i decay with the complex phase φ_{M_1} varied in S4. For both decays the variation of φ_{M_1} is again very small, since both neutralinos are largely determined by μ (for the parameters chosen as in Tab. 3). The full one-loop corrections are the same as for the left plot.

Overall, for the neutral Higgs decays to a neutralino pair we observed, again as expected, an increasing decay width $\propto m_{h_i}$.¹² The full one-loop corrections reach a level of $10 - 20\%$ for decay widths of $\mathcal{O}(1 \text{ GeV})$. The variation with φ_{M_1} is found largest in cases where the \mathcal{CP} -nature of the decay depends strongly on the phase, there then changes by a factor of 5 or more can be observed.

¹² Exceptions are the $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ decay (see the upper left plot of Fig. 15), since m_{h_1} depends only very weakly on M_{H^\pm} . The next exception are the corrections to the h_2 decay in the left plot of Fig. 18 (red line), due to an accidental interplay of anomalous thresholds with the effects induced by the Higgs mixing matrix $\hat{\mathbf{Z}}$. The final exception can be observed in the corrections to the h_2 decay in the left plot of Fig. 22 (red line), because of the additional 2-loop corrections $|\mathcal{M}_{1\text{-loop}}|^2$ (see the discussion of Fig. 22 above).

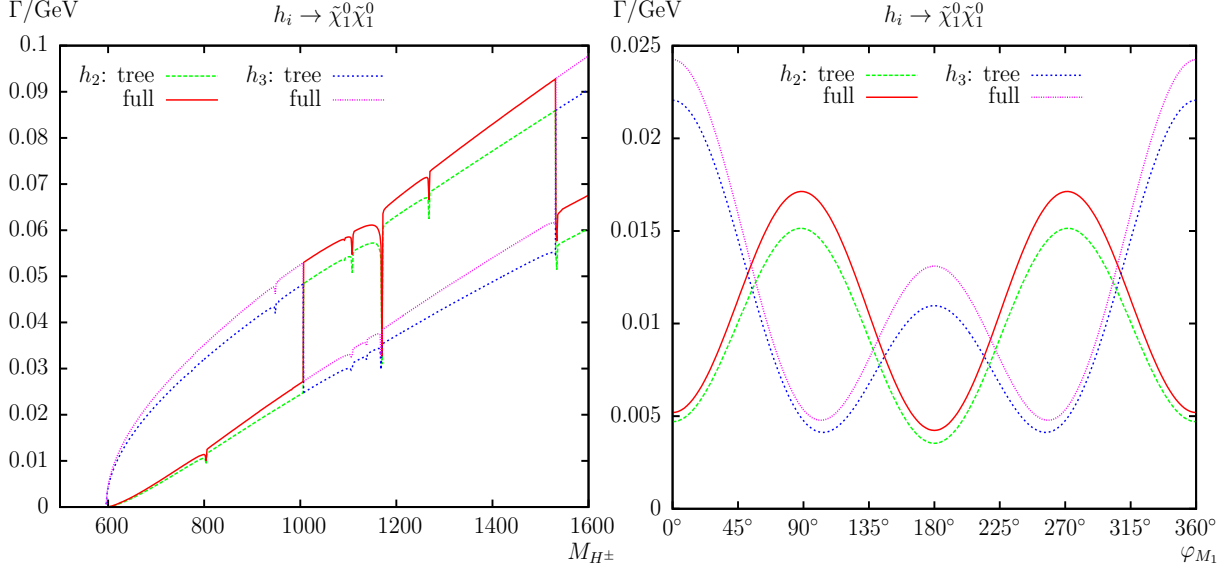


Figure 16: $\Gamma(h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S1 (see Tab. 3).

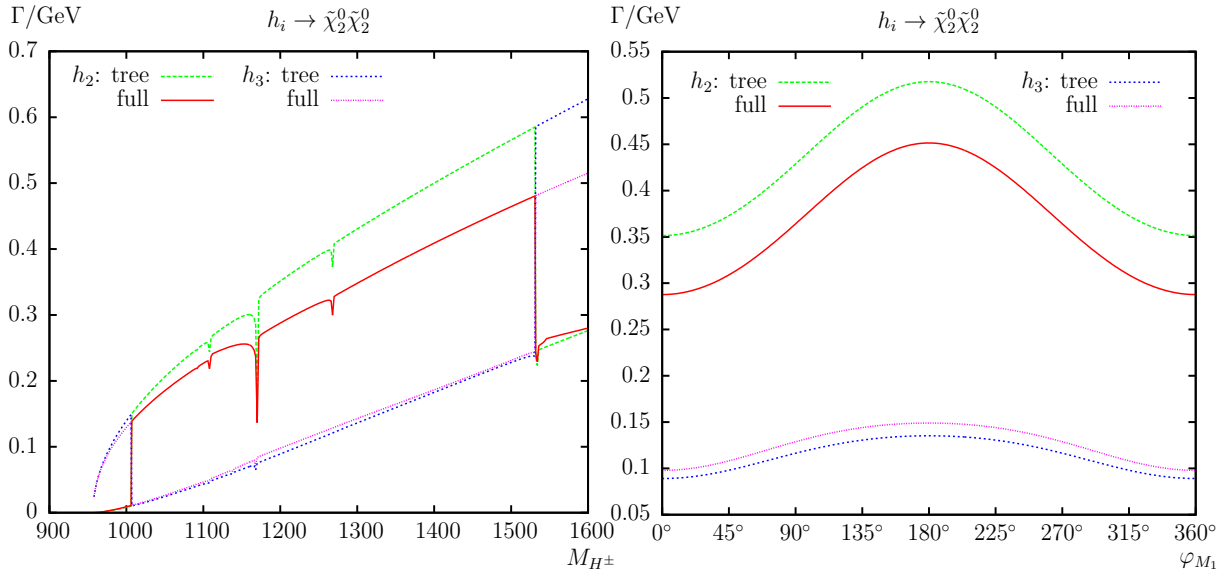


Figure 17: $\Gamma(h_i \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

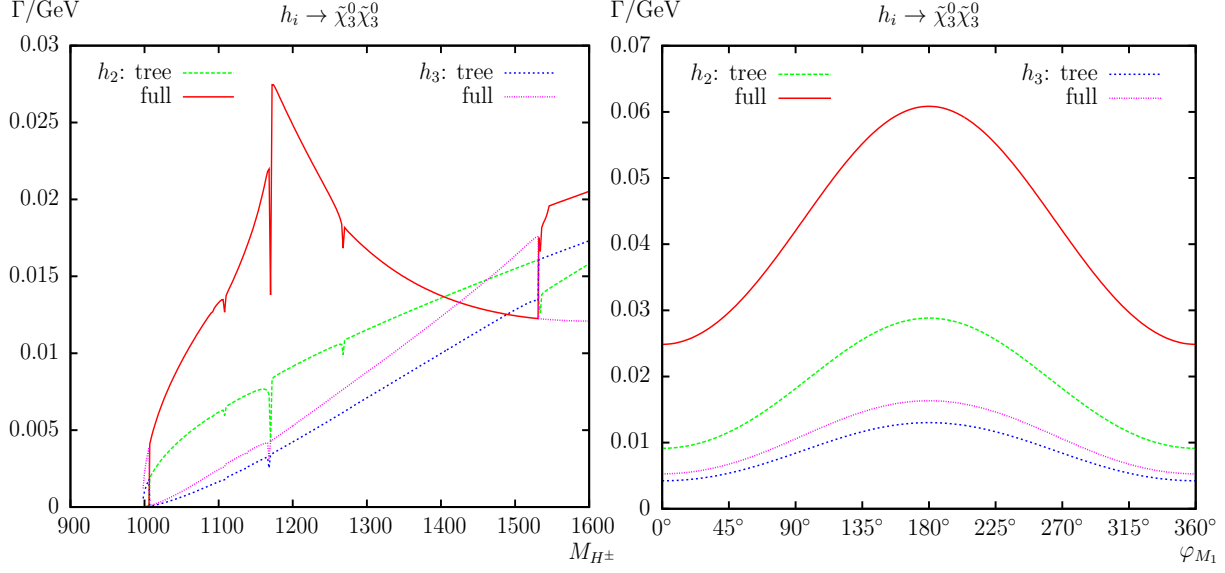


Figure 18: $\Gamma(h_i \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_3^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

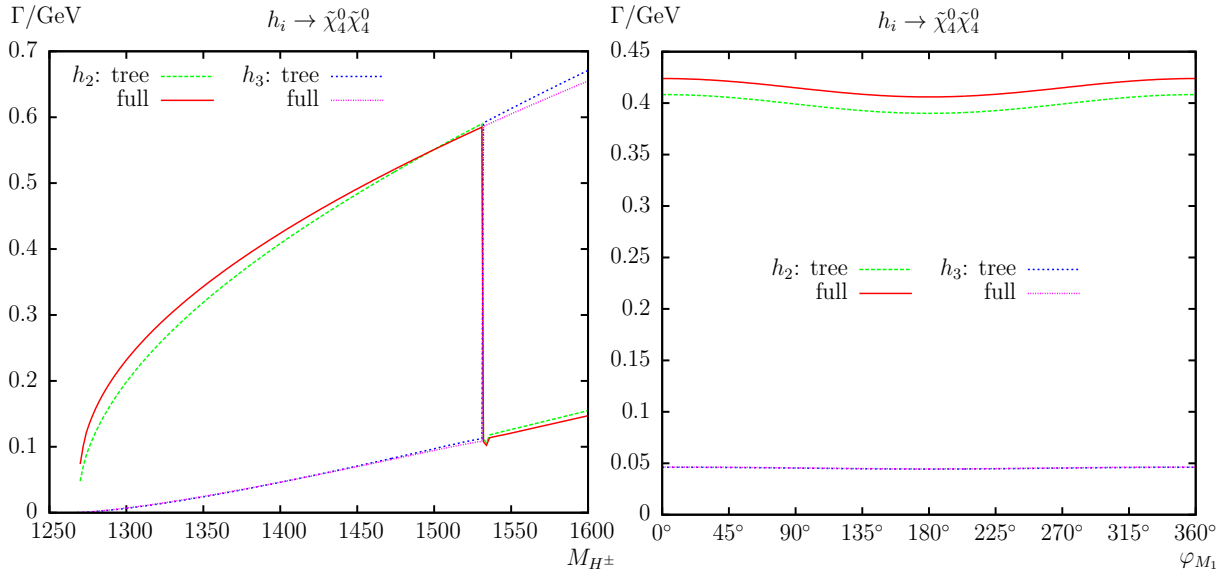


Figure 19: $\Gamma(h_i \rightarrow \tilde{\chi}_4^0 \tilde{\chi}_4^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S5 (see Tab. 3).

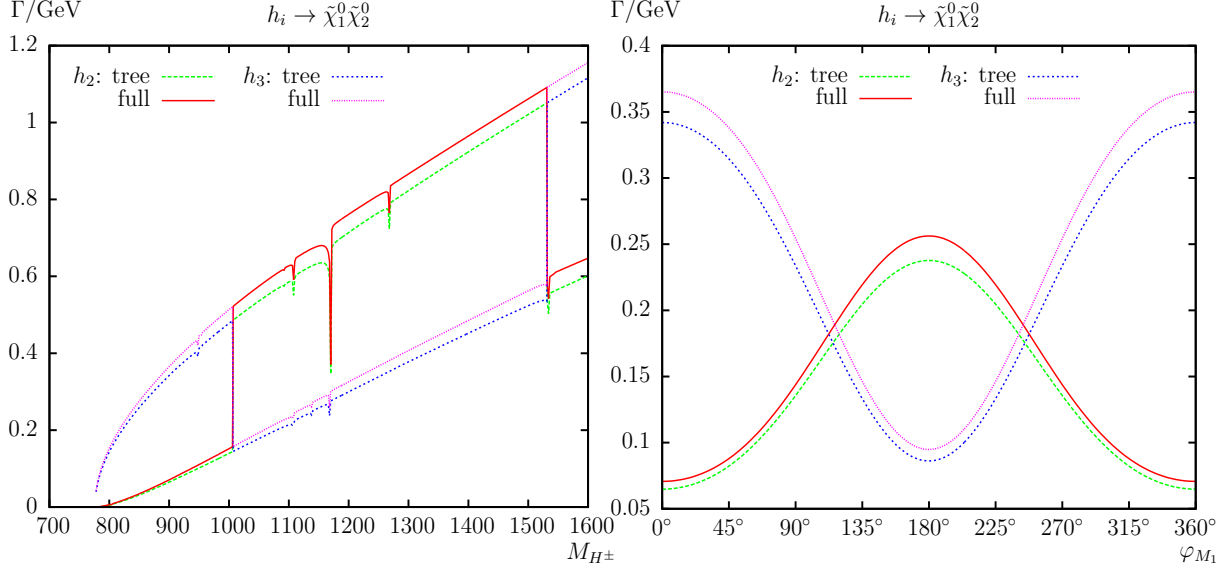


Figure 20: $\Gamma(h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S2 (see Tab. 3).

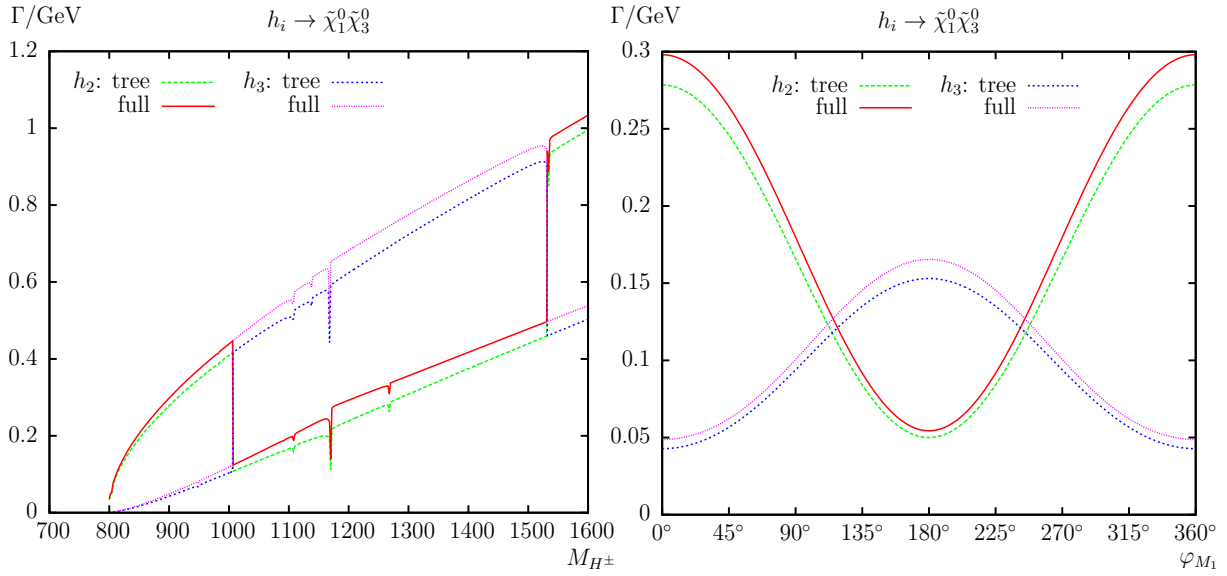


Figure 21: $\Gamma(h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_3^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S2 (see Tab. 3).

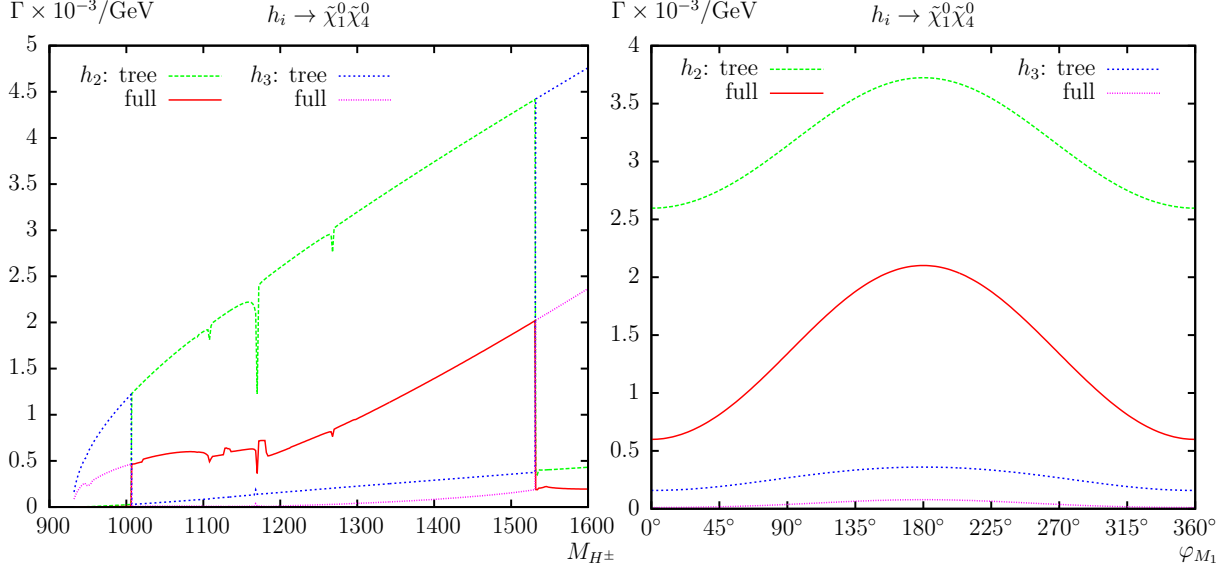


Figure 22: $\Gamma(h_i \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_4^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

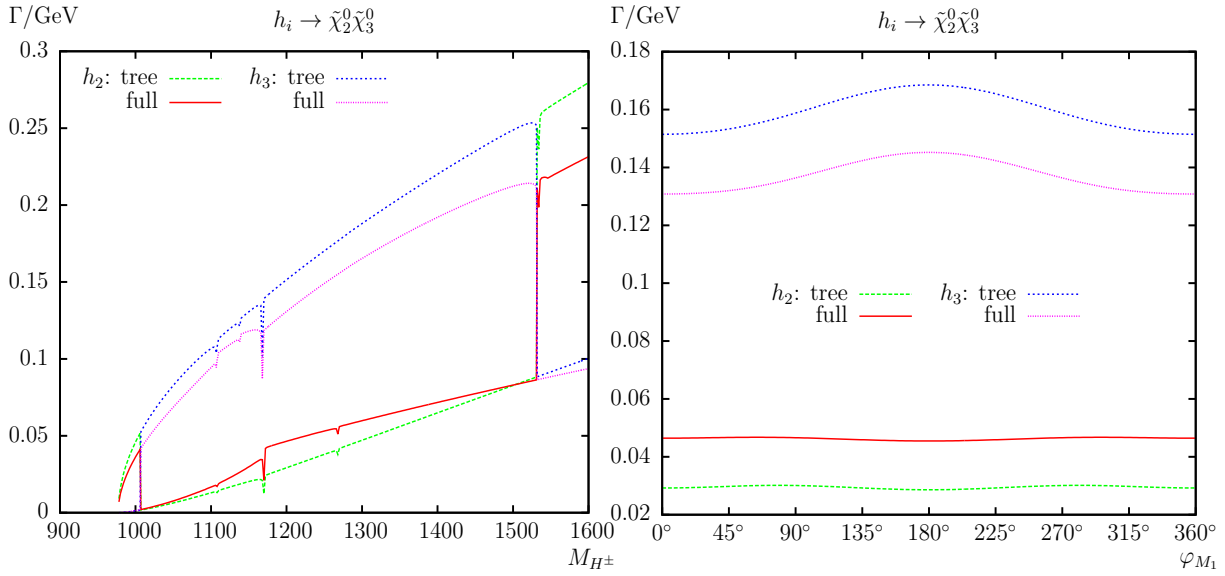


Figure 23: $\Gamma(h_i \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_3^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

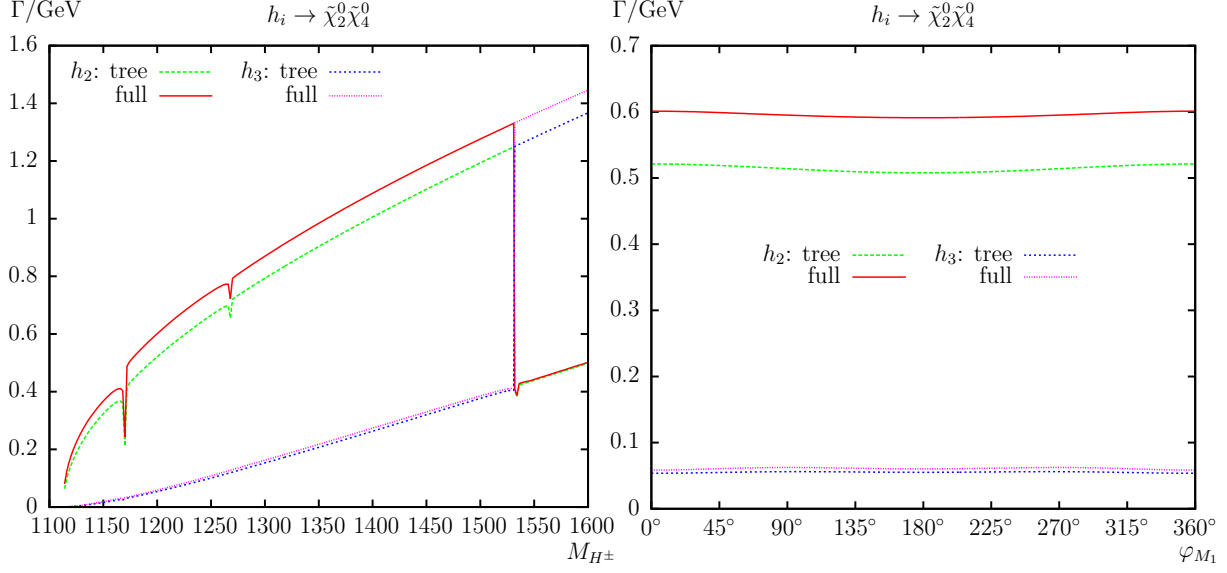


Figure 24: $\Gamma(h_i \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_4^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

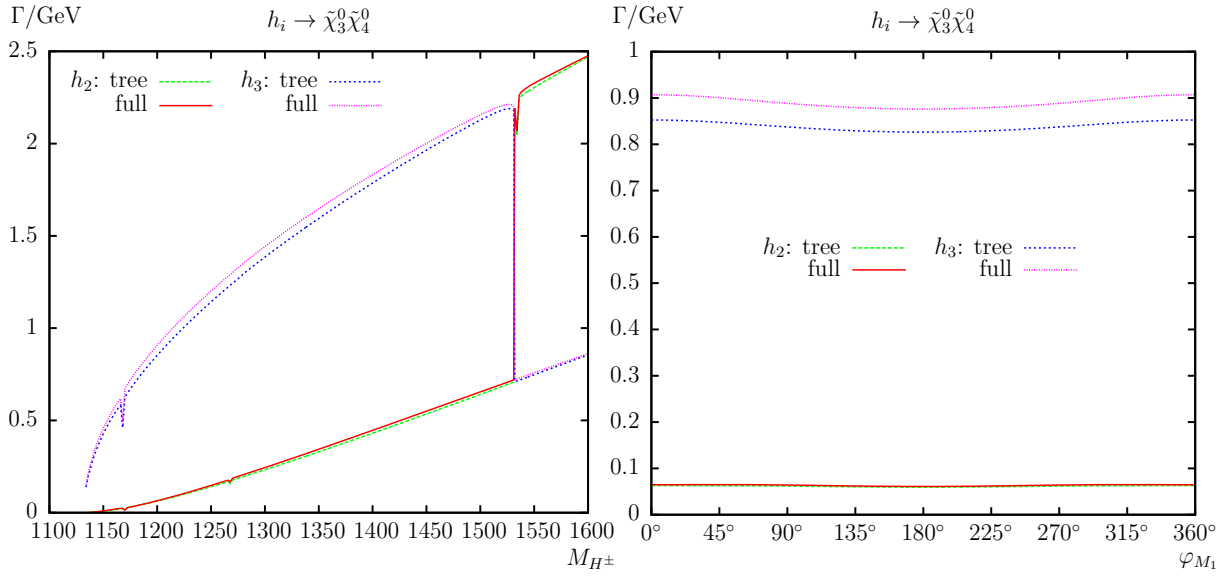


Figure 25: $\Gamma(h_i \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_4^0)$. Tree-level and full one-loop corrected partial decay widths are shown. The left plot shows the partial decay width with M_{H^\pm} varied. The right plot shows the complex phase φ_{M_1} varied with parameters chosen according to S4 (see Tab. 3).

5 Conclusions

We evaluated all partial decay widths corresponding to a two-body decay of the MSSM Higgs bosons to charginos and neutralinos, allowing for complex parameters. In the case of a discovery of additional Higgs bosons a subsequent precision measurement of their properties will be crucial to determine their nature and the underlying (SUSY) parameters. In order to yield a sufficient accuracy, one-loop corrections to the various Higgs-boson decay modes have to be considered. In this work we take another step in the direction of completion of the calculation of *all* two-body decays at the one-loop level in the cMSSM in this stable and reliable renormalization scheme: we calculated all two-body decay modes of the Higgs bosons to charginos and neutralinos in the cMSSM.

The decay modes are given in Eqs. (1) – (3). The evaluation is based on a full one-loop calculation of all decay channels, also including hard and soft QED radiation. We restricted ourselves to a version of our renormalization scheme which is valid for $|M_1| < |M_2|, |\mu|$ and $M_2 \neq \mu$ (where M_1 and M_2 denote the soft SUSY-breaking parameter of the $U(1)$ and $SU(2)$ gauginos, and μ is the Higgs mixing parameter) to simplify the analysis, even though our set-up allows to switch to other parameter regions, possibly implying a different renormalization, see the discussion in Refs. [46–48].

We first reviewed the relevant sectors including some details on the one-loop renormalization procedure of the cMSSM, which are relevant for our calculation. In most cases we follow Ref. [45]. However, in the scalar fermion sector, where we differ from Ref. [45] the relevant details are indicated. We have discussed the calculation of the one-loop diagrams, the treatment of UV and IR divergences that are canceled by the inclusion of (hard and soft) QED radiation. We have checked our result against the literature, and in most cases we have found good agreement, once our set-up was changed to the one used in the existing analyses.

While the analytical calculation has been performed for *all* decay modes to charginos and neutralinos, in the numerical analysis we mostly concentrated on the decays of the heavy Higgs bosons, with $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ being the only channel analyzed for the light neutral Higgs boson. For the analysis we have chosen a parameter set that allows simultaneously a maximum number of two-body chargino/neutralino decay modes. In the analysis either the charged Higgs boson mass or the phase of M_1 has been varied. For M_{H^\pm} we investigated an interval starting at $M_{H^\pm} = 600$ GeV up to $M_{H^\pm} = 1.6$ TeV, which roughly coincides with the reach of the LHC for high-luminosity running as well as an e^+e^- collider with a center-of-mass energy up to $\sqrt{s} \sim 3$ TeV.

In our numerical scenarios we compared the tree-level partial decay widths with the full one-loop corrected partial decay widths. We concentrated on the analysis of the decay widths themselves, since the size of the corresponding branching ratios (and thus the size of their one-loop effects) is highly parameter dependent.

We found sizable corrections of $\sim 10\%$ in many channels, sometimes going up to $\sim 20\%$. Even larger corrections are only found in cases where the tree-level result is accidentally small and thus the decay likely not observable. Corrections at the 10 – 20% level have also been found for the decay $h_1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$, which could constitute an important channel for the determination of the Dark Matter properties in the cMSSM. Consequently, the full one-loop corrections should be taken into account for the interpretation of the searches for

charginos/neutralinos as well as for any future precision analyses of those decays.

The tree-level results, but also the size of the full one-loop corrections often depend strongly on the complex phase analyzed, φ_{M_1} . The one-loop contributions can vary by a factor of ~ 2 as a function of the complex phase. Neglecting the phase dependence could lead to a wrong impression of the relative size of the various decay widths.

In the cases where a decay and its complex conjugate final state are possible, i.e. the charged Higgs decays we have evaluated both decay widths independently. The asymmetries, as a byproduct of our calculation, turn out to be relatively small, at the level of a few percent.

The numerical results we have shown are, of course, dependent on the choice of the SUSY parameters. Nevertheless, they give an idea of the relevance of the full one-loop corrections. Decay channels (and their respective one-loop corrections) that may look unobservable due to the smallness of their decay width in our numerical examples could become important if other channels are kinematically forbidden. Following our analysis it is evident that the full one-loop corrections are mandatory for a precise prediction of the various branching ratios. The full one-loop corrections should be taken into account in any precise determination of (SUSY) parameters from the decay of (heavy) MSSM Higgs bosons. It is planned to implement the evaluation of the branching ratios of the (heavy) Higgs bosons into the Fortran code `FeynHiggs`, together with an automated choice of the renormalization scheme valid for the full cMSSM parameter space.

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